INVESTIGATION OF CONSTRAINTS IN THERMAL SIMILITUDE

VOLUME I

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FOREWORD

This report has been prepared by the Mechanical Engineering Department, Kansas State University, Manhattan, Kansas as part of U. S. Air Force Contract F 33615-68-C-1017, "Investigation of Constraints in Thermal Similitude."

The work was administered under the direction of Air Force Flight Dynamics Laboratory, Air Force Systems Command, with Mr. Carl J. Feldman as Project Engineer. The work was performed between September 1967 and September 1969, with Dr. P. L. Miller as Principal Investigator.

This report was submitted by the authors 1 September 1969.

This technical report has been reviewed and is approved

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ABSTRACT

The studies described in this report clarify the effects of some of the limitations imposed by the laws of thermal similitude, and determine the thermal modeling laws for a heat pipe.

Solutions were presented for the steady-state temperature distribution and heat transfer in a radiating fin having temperature dependent thermal conductivity. Using these solutions, modeling prediction errors were determined for fin type prototype/model systems with dimensional distortions, with material having temperature dependent thermal conductivity, and with low prototype temperatures. These prediction discrepancies ranged from very small errors to errors in heat transfer rates as high as 75% in a severely distorted model.

The thermal modeling laws for a heat pipe were derived and experimentally verified. It was observed that prototype thermal behavior could be predicted, from model data, to within 10°F over the temperature range tested (140 to 330°F). Heat pipe failure due to capillary failure was also predictable to within ±10%.

A flexible heat pipe was also designed and experimentally tested. Performance was not degraded under conditions of bending.

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LIST OF SYMBOLS

\( a = 2 \cos \gamma / k_1 \delta \)
\( B = 2 \cos \gamma / k_1 \delta \)
\( C = k \) constant in Equation (6)
\( D = 2 \cos \gamma / \delta \)
\( k = \) thermal conductivity, \( \text{btu/(hr sqft F)} \)
\( k_1 = \) a constant in the equation \( k = k_1T^a \)
\( l = \) length, \( \text{ft} \)
\( p = \) temperature gradient
\( q'' = \) heat flux, \( \text{btu/(hr sqft)} \)
\( T = \) absolute temperature, \( ^\circ \text{R} \)
\( x = \) variable distance, \( \text{ft} \)

Greek

\( \alpha = a + 1 \)
\( \beta = a + 5 \)
\( \gamma = \beta / \alpha \)
\( \Gamma = \theta / \theta_m \)
\( \delta = \) film half-thickness, \( \text{ft} \)
\( \epsilon = \) film emittance
\( \theta = \) defined in Equation (2)
\( \xi = \theta / \theta_m \)
\( \sigma = \) Stefan-Boltzmann constant, \( \text{btu/(hr sqft R)} \)
\( \phi = \) defined in Equation (20)
\( \rho = \theta / \theta_L \)
\( \rho = \xi / \theta_L \)

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Subscripts

\[ \text{L} = \text{property value at the outer end of the fin} \]
\[ \text{m} = \text{property value at the minimum fin temperature} \]
\[ \text{o} = \text{property value at the fin root} \]
\[ \text{s} = \text{thermal sink property} \]
SECTION I

INTRODUCTION

Thermal scale modeling is a valuable tool in the design of aerospace systems. Significant savings can be achieved if pre-flight testing can be performed in small facilities with reduced size models. Extensive efforts have been made to derive the basic thermal scaling criteria for transient and steady-state conditions. The general laws of thermal similarity have been derived and some experimental work performed which supports these derivations. There are, however, some exceptions that cause difficulties.

The studies performed in this contract attempted to clarify some of the limitations imposed by the thermal modeling laws. The general areas of investigation include:

1. Determination of the limits of the dimensional ratios between model and prototype for both similar and distorted models.
2. Modeling errors caused by material property variations with temperature, or by large temperature differences between model and prototype.
3. Validity of scaling when prototype temperatures are very low.
4. Validity of scaling active heat transfer devices such as a heat pipe.
5. Effects encountered in complex radiative/conductive interchanges when geometry is distorted in the model.
The first three of these studies were accomplished analytically using an exact analysis of a radiating fin as the prototype/model system. These studies are the subject of Volume 1 of this report.

The fourth study was accomplished both analytically and experimentally by deriving the thermal scaling laws for a heat pipe and applying them to the design of a heat pipe prototype/model system which experimentally verified the results. Incidental to this study was the experimental development of a flexible heat pipe. The flexible heat pipe work was undertaken preliminary to the heat pipe modeling program and served to provide initial guidance and experience in the technology of heat pipes. These studies are the subject of Volume 2 of this report.

The fifth study was not successfully accomplished.
SECTION II

THERMAL SCALE MODELING

The basis for the thermal modeling of a physical system is the maintenance of thermal similitude between model and prototype. Thermal similitude is maintained by preserving the numerical equality of certain dimensionless groups of properties of the prototype and model. These dimensionless groups, or similarity parameters, must contain all the physical quantities which interact to determine the thermal behavior of the system.

The derivation of the similarity parameters has been performed by several authors (1,2,3,4). Perhaps the most useful to this study was the derivation by Miller and Wiebelt (4) who derived the set of similarity parameters given in Table I. The "starred" values represent the model/prototype value of the variable (\( \tau^* = \frac{\tau_{\text{model}}}{\tau_{\text{prototype}}} \)), etc.

The assumptions necessary for the derivation of this set of parameters are as follows:

1. Steady state heat transfer.
2. System shape is a solid cylinder.
3. Surface radiation properties of model and prototype are uniform and equal.
4. There is no simulation of solar radiation.
5. Model and prototype are made of the same material.
6. Material thermal conductivity varies with temperature according to the relation (with \( a = 0 \) conductivity is constant).
### TABLE 1

**Similarity Parameters for Solid Cylindrical Systems**

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<th>Parameter</th>
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<th>Method 2</th>
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<tr>
<td>$q^*$</td>
<td>$\frac{\mathcal{B}^<em>(7-\alpha)}{\mathcal{L}^</em>(5+\alpha)} \frac{1}{\beta-\alpha}$</td>
<td>$L^4$</td>
<td>$\frac{\mathcal{R}^<em>}{\mathcal{L}^</em>} \frac{1}{\beta-\alpha}$</td>
</tr>
<tr>
<td>$T^*$</td>
<td>$\left[ \frac{\mathcal{R}^<em>}{\mathcal{L}^</em>} \right] \frac{1}{\beta-\alpha}$</td>
<td>$1$</td>
<td>$\frac{\mathcal{R}^<em>}{\mathcal{L}^</em>} \frac{1}{\beta-\alpha}$</td>
</tr>
<tr>
<td>$L^*$</td>
<td>$- - -$</td>
<td>$\frac{1}{R^2}$</td>
<td>$- - -$</td>
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</table>
The method 1 parameters take into account property variations with temperature and permit the prototype and model to be at different temperatures if desired. The method 2 parameters are a special case of method 1 where the model is distorted \( R^* = L^* \) so that model and prototype temperatures are the same, and method 3 parameters are the special case, with \( a = 0 \), of constant thermal conductivity.

As described, the thermal modeling parameters presented in Table I are written for a rather restricted set of circumstances. It is quite possible to present a much more general set of modeling parameters. Unfortunately, it has not been possible to utilize these general parameters to obtain a general method of error prediction for thermal modeling. However, through a study of the prediction errors of the more restricted fin systems it will be possible to obtain a better insight into the errors involved in the use of the general modeling parameters.

Section III of this report includes a derivation of equations which describe the steady state temperature distribution and heat transfer rates in a radiating fin. These equations are used in Section IV to determine errors involved in the thermal modeling of a fin using the parameters listed in Table I.
SECTION III

STEADY-STATE ANALYSIS OF A RADIATIVE FIN HAVING TEMPERATURE-DEPENDENT CONDUCTIVITY

1. INTRODUCTION

This section presents a steady-state analysis of radiating fins which have uniform cross section, constant emittance, temperature-dependent thermal conductivity, and either zero or non-zero thermal sink boundary conditions. The analysis is presented as a series of eight different cases of boundary conditions applied to the fin.

An analysis not previously presented in the literature, even for a fin with constant thermal conductivity, is presented as Case VII. Case VI describes a fin of finite length which is radiating to a non-zero sink and which has fixed but unequal temperatures on the ends. This solution is particularly useful in calculating energy losses from a thermal model in a space simulation chamber to its instrumentation and support wires.

Steady-state analyses of radiating fins with constant thermal conductivity have been presented by several authors [6, 7]. One previous paper considered a finite length fin with insulated end and a conductivity which varied linearly with temperature, similar to Case VI of this article [8].

2. ANALYSIS

Let us consider the steady-state behavior of a fin of variable thermal conductivity having a uniform root temperature of $T_r$, protruding into an evacuated "black" space having an effective uniform radiating temperature of $T_a$. Figure 1 shows the dimensions of a typical uniform cross section rectangular fin, infinitely wide in the "z" direction. We consider only a unit width portion of the fin so that edge effects may be neglected. We
FIGURE 1. RADIATING FIN
further consider the fin to be "thin" relative to the length and width so that the heat transfer is, for practical purposes, one dimensional. An energy balance on an elemental section of the fin results in the following differential equation describing the temperature distribution in the fin as a function of distance from the root.

\[ \frac{d}{dx}(k \frac{dT}{dx}) - \frac{C_p}{\epsilon}(T^m - T^*) = 0 \]  

(1)

This analysis assumes there is no radiant interchange between the fin and its base and that there is no external source of radiant flux such as a solar simulator. If, however, these fluxes are present and are not functions of \( x \), they may be included in the \( T^* \) term by an appropriate additive factor.

Equation (1) applies also to a fin of circular cross-section (pin-fin) if one-fourth of the diameter of the fin is used instead of the rectangular fin half-thickness (4).

For convenience, let us define a new variable, \( \Theta \), as

\[ \Theta = \frac{1}{k_1} \int_0^T k \, dT \]  

(2)

where \( k_1 \) is a constant, then

\[ k_1 \frac{d\Theta}{dx} = k \frac{dT}{dx} \]  

(3)

The variable \( \Theta \) obviously depends upon the relationship between the thermal conductivity and the temperature. One relationship which has proved useful over a reasonable range of temperatures, for various metals, is of the form described by the equation

\[ k = k_1 T^a \]  

(4)

where \( k_1 \) and \( a \) are constants [4,5].
By substitution of Equation (4) into Equation (2) a relationship between \( \theta \) and \( T \) is obtained.

\[
T = (\alpha \theta)^{\frac{1}{a}} \quad \text{as} \quad 0
\]

where \( a = a + 1 \)

Equation (1) then becomes

\[
\frac{d\theta}{dx} = \frac{c_0 (\gamma-1)}{k_1 \delta} \left[ \gamma (\gamma-1) - \delta (\gamma-1) \right] = 0
\]

where \( \gamma = \frac{a + 5}{a + 1} \)

A first indefinite integration of this equation leads to

\[
\frac{d\theta}{dx} = \frac{\theta}{a(x - \delta_0 (\gamma-1) + C)}^{\frac{1}{2}}
\]

where \( B = \frac{2c_0 \gamma}{k_1 \delta} \)

\( a = a + 5 \)

Equation (7) may be integrated from \( \theta_0 \) to \( \theta \) as \( x \) goes from 0 to \( x \) to give

\[
x = \frac{\theta}{B(x - \delta_0 (\gamma-1) + C)^{\frac{1}{2}}}
\]

(Equation 8) requires the application of a boundary condition to determine the constant \( C \) and the proper sign before it may be integrated.

The situation to be presented as Case I allows Equation (8) to be formally integrated, and for Case II, a solution in terms of complete and incomplete Beta functions has been given by Chen [6]. For most cases to be considered in this article, however, there is apparently no closed form solution to Equation (8). The use of digital computers for numerical integration, together with, in some cases, an iteration technique, makes possible a
solution to the various forms of Equation (8) which result from the application of boundary conditions from a particular problem.

Table I presents a summary of eight sets of boundary conditions and the corresponding values of the constant C to be substituted into Equation (8) before integration. For all cases, it is required that the fin root temperature $T_0$ be greater than $T_m$, and in Cases V, VI and VIIa it is required that $T_m < T_K < T_0$.

Case I

Case I considers an infinite length fin radiating to a zero temperature sink. For this case the temperature and the temperature gradient must both approach zero as x approaches infinity. For these boundary conditions the constant C is zero and Equation (8) may be integrated. The result is, in terms of $T$ and $T_0$,

$$x = \left( \frac{2k_0\delta \eta}{cT(3-\alpha)^2} \right) \left[ \frac{1}{2} \left( \frac{T}{T_0} \right)^{\frac{3-\alpha}{2}} - \frac{3-\alpha}{2} \right]$$  \hspace{1cm} (a<3) \hspace{1cm} (9)

The heat flux, per unit fin width, at the fin root is, in terms of $T$ and $T_0$,

$$q_0'' = \frac{G}{\sqrt{T_0}}$$ \hspace{1cm} (10)

where $G = \frac{2\pi k_0}{\delta}$

Case II

For Case II, the fin is of finite length with the end insulated. The temperature at the end of the fin, $T_L$, is unknown and must be determined by an iteration procedure. By first applying the boundary conditions from Table I to Equation (7) to determine C, Equation (8) may be written as follows.
\[ x = -\int_{\theta_0}^{\phi} \frac{d\theta}{B(\theta^\gamma - \phi_0^\gamma)^{1/2}} \]  \hspace{1cm} (11)

Let us define \( \phi \) as follows, recognizing that \( \phi_0 \) has a fixed value for a particular problem.

\[ \phi = \frac{\theta}{\phi_0} \]  \hspace{1cm} (12)

By substitution of this relation into Equation (11), inverting the limits and expanding the integral, there results

\[ x = \left( \frac{1}{B \phi_0^{(\gamma - 2)/2}} \right)^{1/2} \left[ \int_{\phi_0}^{\theta} \frac{d\theta}{(\theta^\gamma - 1)^{1/2}} - \int_{\phi_0}^{\phi} \frac{d\theta}{(\theta^\gamma - 1)^{1/2}} \right] \]  \hspace{1cm} (13)

By noting that \( \phi = \phi_0 \rightarrow 1 \) as \( x \rightarrow L \), Equation (13) may be reduced to

\[ L = \left( \frac{1}{B \phi_0^{(\gamma - 2)/2}} \right)^{1/2} \int_{\phi_0}^{\phi} \frac{d\theta}{(\theta^\gamma - 1)^{1/2}} \]  \hspace{1cm} (14)

The integral in Equation (14) may be evaluated numerically without great difficulty. Caution must be exercised in evaluating the function

\[ f(\theta) = \frac{1}{(\theta^\gamma - 1)^{1/2}} \]  \hspace{1cm} (15)

near \( \theta = 1 \) so that sufficient accuracy is obtained.

Once \( \phi_0 \) is determined from Equation (14), the temperature at the end of the fin, \( T_L \), is known and \( x \) may be determined for any desired temperature between \( T_0 \) and \( T_L \) from Equation (13). The heat flux, per unit fin width, at the fin root may be shown to be, in terms of \( T_0 \) and \( T_L \).

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\[ q_0'' = \left[ 2c_0^2 \left( \gamma - 1 \right) \right] \frac{1}{k_1} \]  \hspace{1cm} (16)

**Case III**

The constant \( C \) for the fin of finite length with \( T_o = 0 \) and a radiating end, Case III, may be determined by equating the radiative flux to the conductive flux at the end of the fin, thus obtaining an expression for the thermal gradient at \( x = L \).

\[ q_L'' = -k_1 \frac{\partial \theta}{\partial x} \bigg|_{x=L} = \frac{2c_0}{k_1} \left( \frac{\theta}{\gamma - 1} \right) \]  \hspace{1cm} (17)

Using this expression and Equation (7), the result is:

\[ C = -2c_0 \left( \frac{\theta}{\gamma - 2} \right) \frac{1}{k_1} \]  \hspace{1cm} (18)

By making the substitution of \( C \) into Equation (8), and the additional substitutions of

\[ \psi = \frac{\theta}{\theta_L} \]  \hspace{1cm} (19)

\[ \tau = \left( \frac{1 - \frac{2c_0 \theta}{k_1 \theta_L}}{k_1} \right)^{\frac{1}{\gamma - 2}} \]  \hspace{1cm} (20)

and expanding the integral, there results

\[ x = \left[ \frac{(2-\gamma)}{2\gamma \theta_L} \right] \left( \frac{\psi}{\psi_L} \right) \int_{\psi_0}^{\psi} \left( \frac{d\psi}{(\psi - 1)^{1/2}} - \int_{\psi}^{\psi_0} \frac{d\phi}{(\phi - 1)^{1/2}} \right) \]  \hspace{1cm} (21)

By noting that as \( x = L, \psi \rightarrow \frac{1}{\tau} \) and \( \phi \rightarrow \theta_L \), Equation (21) may be reduced to

\[ L = \left[ \frac{(2-\gamma)}{2\gamma \theta_L} \right] \left( \frac{\psi}{\psi_L} \right) \int_{\psi}^{\psi_0} \left( \frac{d\psi}{(\psi - 1)^{1/2}} - \int_{\psi}^{\psi_0} \frac{d\phi}{(\phi - 1)^{1/2}} \right) \]  \hspace{1cm} (22)

from which \( \theta_L \) and thus \( \tau \) may be determined by iteration. Once \( \theta_L \) is determined \( T_L \) is known and the location \( x \) of any temperature between \( T_0 \) and
$T_L$ may be determined from Equation (21).

The heat flux, per unit fin width, at the root of the fin may be shown to be

$$h_0 = \left[ 6 \left( \frac{1}{\alpha} - \frac{r_0}{T_L} \right) + (c\alpha) \right]^2 \frac{1}{2}$$

(23)

**Case IV**

The temperature distribution in a fin radiating to a zero temperature sink and having both end temperatures fixed at unequal values ($T_0 > T_L$) is considered next. The gradient at $x = L$ is fixed by the boundary conditions but is unknown and may be either positive or negative, depending upon the fin properties and the end temperatures.

Applying the condition $\theta_m = 0$, Equation (7) may be written as

$$\frac{d\theta}{dx} \bigg|_{x=L} = p_L = \pm (B0_L^Y + C)^2$$

(24)

from which

$$C = \frac{p_L^2 - B0_L^Y}{P_L}$$

(25)

Equation (8) may then be written as

$$x = \pm \left[ \frac{1}{B0_L^Y} \right] \left[ \theta - \left( \frac{p_L^2}{B0_L^Y} \right)^{1/2} \right]^{1/2} \left( \theta^2 - 1 + \frac{p_L^2}{B0_L^Y} \right)^{1/2}$$

where $\theta = \frac{\theta}{\theta_L}$

(26)

(27)

We shall consider three special cases of boundary conditions for which this equation must be solved.

Figure 2 depicts the temperature distribution in a radiating fin with all problem conditions fixed except fin length. In Figure 2a the fin is...
Figure 2. Fin Temperature Distribution
short enough so that for any \( x \), \( \frac{d\theta}{dx} < 0 \). Figure 2b depicts the precise length such that \( \frac{d\theta}{dx} = 0 \), which is identical to Case II. As the fin is progressively lengthened, the temperature passes through some minimum value less that \( \theta_L \) and begins to increase with \( x \) rather than decrease. These three cases will be called Case IVa, IVb, and IVc respectively.

To determine which case is actually present under specified conditions, the following procedure is used:

1. Solve Case II, given \( \theta_0 \) and \( \theta_L \) for the length of an insulated end fin (\( L_2 \)).

2. If \( L_2 \) is greater than the specified length of the fin for the Case IV problem (\( L_4 \)), then \( \frac{d\theta}{dx} \) is everywhere negative, and Equation (26) may be written

\[
L = \left( \frac{1}{\ln\left(\frac{\gamma-1}{\gamma-2}\right)} \right)^{\frac{1}{2}} \left( \frac{1}{1 \left\{ \frac{\gamma-1}{\gamma-2} \right\} + \frac{p_L^{1/2}}{Bf_{L^2}} \right)^{1/2} \theta_0 \frac{d\theta}{dx} \right)
\]  

(28)

This equation may be used with an integration-iteration process to solve for the unknown \( p_L \), and then Equation (26) is used to solve for the location \( x \) of any desired temperature \( \theta_0 < \theta < \theta_L \). The heat flux at the root of the fin is

\[
q_0'' = (\theta_0'' - T_L^\theta + \frac{1}{p_L^{1/2}})
\]

(29)

The heat flux at the outer end of the fin is

\[
q_L'' = k_L p_L
\]

(30)

3. If \( L_2 = L_4 \) then Case II exists.
4. If \( L_2 < L_4 \) then a minimum temperature exists at some location in
the fin. At that minimum, \( \frac{dO}{dx}\big|_{x=x_m} = 0 \). Using Equation (7) and
giving the temperature at the minimum the subscript \( m \), then
\[
C = -\beta_0 \gamma m
\]  
(31)
Introducing the limits to equation (8) and expanding it to
include the effects of both positive and negative gradients,
there results
\[
L = \left[ \frac{1}{\beta_0 (\gamma - 2)} \right]^{\frac{1}{2}} \int_{1}^{\Gamma_0} \frac{d\Gamma}{(\Gamma - 1)^{\frac{1}{2}}} + \int_{1}^{\Gamma_L} \frac{d\Gamma}{(\Gamma - 1)^{\frac{1}{2}}}
\]  
(32)
where
\[
\Gamma = \frac{\theta}{\theta_m}
\]  
(33)
The unknown \( \theta_m \) may be solved for by an integration-iteration
process as previously described. Once \( \theta_m \) is known, the location
of the minimum temperature is determined from
\[
x_m = \left[ \frac{1}{\beta_0 (\gamma - 2)} \right]^{\frac{1}{2}} \int_{1}^{\Gamma_0} \frac{d\Gamma}{(\Gamma - 1)^{\frac{1}{2}}}
\]  
(34)
and then the temperature distribution along the fin may be
determined from the appropriate equation below.

for \( 0 < x < x_m \)
\[
x = x_m + \left[ \frac{1}{\beta_0 (\gamma - 2)} \right]^{\frac{1}{2}} \int_{1}^{\Gamma} \frac{d\Gamma}{(\Gamma - 1)^{\frac{1}{2}}}
\]  
(35)

for \( x_m < x < L \)
\[
x = x_m + \left[ \frac{1}{\beta_0 (\gamma - 2)} \right]^{\frac{1}{2}} \int_{1}^{\Gamma} \frac{d\Gamma}{(\Gamma - 1)^{\frac{1}{2}}}
\]  
(36)
The heat flux at the root of the fin may be determined from

\[ q''_0 = \left[ G(T_0^B - T_0^S) \right]^{\frac{1}{2}} \]  \hspace{1cm} (37)

and the heat flux at the outer end may be written

\[ q''_L = \left[ G(T_L^B - T_L^S) \right]^{\frac{1}{2}} \]  \hspace{1cm} (38)

**Case V**

For the case of infinite length fin where \( T_0 \neq 0 \), let us define a new variable \( \zeta \) as

\[ \zeta = \frac{\theta}{\theta_m} \]  \hspace{1cm} (39)

and use the boundary conditions \( \theta \to \theta_m \) and \( \frac{d\theta}{dx} \to 0 \) as \( x \to \infty \) in Equation (7) to obtain:

\[ C = -B \theta_0^\gamma (1 - \gamma) \]  \hspace{1cm} (40)

Equation (8) may then be reduced to

\[ x = \left[ \frac{1}{B \theta_0^\gamma (\gamma - 2)} \right]^{\frac{1}{2}} \int_0^\zeta \frac{d\xi}{[\xi^\gamma - 1 - \gamma(\xi - 1)]^{\frac{1}{2}}} \]  \hspace{1cm} (41)

Both \( \theta_m \) and \( \zeta_0 \) are known so that this equation may be solved numerically for the location \( x \) of any temperature between \( T_0 \) and \( \zeta_0 \). The heat flux at the base may be shown to be, in terms of \( T_0 \) and \( T_\infty \)

\[ q''_0 = \left[ G[T_0^B - (1 - \gamma)T_\infty^S] \right]^{\frac{1}{2}} \]  \hspace{1cm} (42)

**Case VI**

Case VI is for a fin of finite length, radiating to a non-zero temperature sink and having the free end insulated. By letting

\[ \xi = \frac{\theta}{\theta_m} \]  \hspace{1cm} (43)

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and using the boundary condition \( \frac{d\xi}{dx} |_{x=L} = 0 \) we obtain the constant of integration \( C \) in terms of an unknown \( \xi_L \). The resulting relation is given in Table I. This result may be substituted into Equation (8) to obtain

\[
X = \left[ -\frac{1}{B_0 (\gamma-2)} \right]^{1/2} \int_{\xi}^{\xi_0} \frac{d\xi}{\left[ \xi^\gamma - \xi_L^\gamma - \gamma (\xi - \xi_L)^{\gamma/2} \right]^{1/2}}
\]

(44)

The limiting conditions are \( \xi = \xi_L \) as \( x \to L \), with which we obtain

\[
L = \left[ -\frac{1}{B_0 (\gamma-2)} \right]^{1/2} \int_{\xi_L}^{\xi} \frac{d\xi}{\left[ \xi^\gamma - \xi_L^\gamma - \gamma (\xi - \xi_L)^{\gamma/2} \right]^{1/2}}
\]

(45)

The unknown \( \xi_L \) may be determined from Equation (45) by a numerical iterative technique. The second law of thermodynamics requires that \( \xi_L \) exceed unity.

\[
q^a_L = \frac{1}{2} (G [\xi_L^\gamma - \gamma T_0^\gamma (T_0^a - T_L^a)])
\]

(46)

Case VII

Case VII presents a more difficult problem and will be considered in three separate parts. First, however, we derive the temperature distribution equation by again applying the boundary conditions to Equations (7) and (8). The temperature and the temperature gradient at the end of the finite length \( f_m \) are not zero and the gradient is unknown. Equation (7) may be used to obtain a relation for the constant \( C \). By defining:

\[
\xi = \frac{\xi}{\xi_0}
\]

(47)

the result is

\[
C = \frac{2}{\xi_L^2} - \frac{2}{\xi_0^2} \xi_L^2 - \gamma \xi_L
\]

(48)
Equation (48) then may be written

\[
x = \left[ \frac{1}{\text{Re}_m^{1/2}} \right]^{1/2} \int_{\xi}^{\xi_0} \frac{d\xi}{\left( \xi^2 - \xi_0^2 - \gamma(\xi - \xi_L) + \frac{p_L^2}{\text{Bo}_m^2} \right)^{1/2}} \quad (49)
\]

The first situation we shall consider is for $0 > \theta_L$, designated Case VIIa. The variable $\xi$ ranges in value from greater than unity to less than unity as the temperature decreases along the fin. When the fin temperature is above $T_m$, the fin loses heat to the surroundings and the opposite is true when the fin temperature is below $T_m$.

At some value of $x$ along the fin there is an inflection point in the temperature vs. length curve, at which point the absolute value of the slope of the curve is a minimum, but not zero. Temperature is thus a single-valued function of distance. For this situation, the integral limits of Equation (48) are $\xi_0$ and $\xi_L$ at $x = 0$ and $x = L$, respectively.

\[
L = \left[ \frac{1}{\text{Bo}_m^{1/2}} \right]^{1/2} \int_{\xi}^{\xi_L} \frac{d\xi}{\left( \xi^2 - \xi_L^2 - \gamma(\xi - \xi_L) + \frac{p_L^2}{\text{Bo}_m^2} \right)^{1/2}} \quad (50)
\]

The value of $p_L$ may thus be determined, by an iteration process, from Equation (50). Once $p_L$ is known, Equation (49) may be used to determine the location of any temperature which exists along the fin.

The heat flux at the fin root, per unit of fin width, may be written as follows:

\[
q''_0 = \left[ G(T''_C - T''_L - T''_0 \gamma(T''_0 - T''_L) + k_1 n_L^2)^{1/2} \quad (51)
\]

For the situation of Case VIIb where $\theta_L = \theta_m$, the governing Equations (48) through (51) are charged only to the extent that $\xi_L = 1$. The procedure

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for a solution is the same as previously discussed under Case VIIa. This situation approaches Case V if the fin is long and the conductance is low.

If \( \theta_L > \theta_w \), Case VIIr, the situation becomes more complex because the temperature distribution along the fin may be double-valued in \( x \). Figure 2 shows the situation of an imaginary fin of fixed thermal properties but various lengths. In Figure 2a, the fin is short enough that for any \( x \), 
\[
\frac{d\theta}{dx} < 0.
\]
Figure 2b depicts the precise length such that 
\[
\frac{d\theta}{dx} \bigg|_{x=L} = 0,
\]
is identical to Case VI. As the fin is progressively lengthened the temperature passes through some minimum value less than \( \theta_L \) and begins to increase again.

The series of steps which will lead to a solution to this problem are as follows:

1. Solve Case VI, given \( \theta_L \), for the length of an insulated end fin \( L_0 \).
2. If \( L_0 \) is greater than the specified length of the fin for the Case VII problem \( L_f \), then the situation in Figure 2a exists and Equations (49) and (50) may be used to determine the temperature distribution.
3. If it happens that \( L_0 = L_f \) then Case VI exists and Equations (44) and (46) are used to determine the temperature and heat flux.
4. If \( L_0 < L_f \) then the fin temperature goes below \( \theta_L \) somewhere along the fin. The location of the point of minimum temperature is unknown but the temperature gradient at that value of \( x \) must be zero. Using Equation (7) and giving the temperature at that point a subscript of \( w \), then
\[ C = -\beta_0 \left( \frac{Y}{l_m} - \frac{Y}{l_m} \right) \]  

Introducing the limits to Equation (8) and expanding it to include the effects of both positive and negative gradients, the results:

\[ L = \frac{1}{\beta_0 (Y-2)} \left( \frac{\xi_0}{l_m} \right)^{1/2} \int_{l_m}^{\xi_0} \frac{d\xi}{\left( \xi' - \xi_m - \gamma (\xi - \xi_m) \right)^{1/2}} \]

\[ + \int_{l_m}^{\xi_L} \frac{d\xi}{\left( \xi' - \xi_m - \gamma (\xi - \xi_m) \right)^{1/2}} \]

where:

\[ \xi_L > \xi_m > 1 \]

Equation (53) must be solved for \( \xi_m \) by iteration. Once \( \xi_m \) is determined, \( x_m \), the location of the minimum temperature may be determined from:

\[ x_m = \frac{1}{\beta_0 (Y-2)} \left( \frac{\xi_0}{l_m} \right)^{1/2} \int_{l_m}^{\xi_0} \frac{d\xi}{\left( \xi' - \xi_m - \gamma (\xi - \xi_m) \right)^{1/2}} \]

The temperature distribution along the fin may then be determined from the appropriate equation below.

\[ \begin{aligned} &\text{for } 0 < x < x_m \\
&x = \frac{1}{\beta_0 (Y-2)} \left( \frac{\xi_0}{l_m} \right)^{1/2} \int_{l_m}^{\xi_0} \frac{d\xi}{\left( \xi' - \xi_m - \gamma (\xi - \xi_m) \right)^{1/2}} \end{aligned} \]  

\[ \begin{aligned} &\text{for } x_m < x < L \\
&x = x_m + \frac{1}{\beta_0 (Y-2)} \left( \frac{\xi_0}{l_m} \right)^{1/2} \int_{l_m}^{\xi_0} \frac{d\xi}{\left( \xi' - \xi_m - \gamma (\xi - \xi_m) \right)^{1/2}} \end{aligned} \]

For Case VIIc, the equation for the heat flux at the fin root, per unit fin.
width, is
\[ q_o = \frac{1}{6}(\gamma \tau_o^2 - \tau_n^2) \left( \gamma \tau_o^2 - \tau_n^2 \right)^{\frac{1}{2}} \]  \hspace{1cm} (57)
and the heat flux at the outer end of the fin is
\[ q_L = -\frac{1}{6}(\gamma \tau_L^2 - \tau_m^2) \left( \gamma \tau_L^2 - \tau_m^2 \right)^{\frac{1}{2}} \]  \hspace{1cm} (58)

Case VIII

This case is similar to Case VI except that the end of the fin is radiating instead of being insulated. As before, let
\[ \xi = \frac{\theta}{\theta_m} \]  \hspace{1cm} (59)
and use the radiating boundary condition at the end of the fin
\[ \frac{d\xi}{dx} \bigg|_{x=L} = -\frac{c_0(a_0^\gamma)}{a_L} \left( \xi_L^{(y-1)} - 1 \right) \]  \hspace{1cm} (60)
to obtain the constant \( C \) in terms of an unknown \( \xi_L \). The equation for \( C \) is given in Table I.

This result may be substituted into Equation (9) to obtain
\[ x = \left[ \frac{1}{8(L-1)} \right]^{\frac{1}{2}} \int_0^L \frac{\xi d\xi}{\xi^2 - \xi_L^2 - (\xi_L^{(y-1)} - 1)^2 \left( \xi_L^{(y-1)} - 1 \right)^2} \]  \hspace{1cm} (61)
The limiting conditions are \( \xi \to \xi_L \) as \( x \to L \), with which we obtain
\[ L = \left[ \frac{1}{8(L-1)} \right]^{\frac{1}{2}} \int_L^\infty \frac{\xi d\xi}{\xi^2 - \xi_L^2 - (\xi_L^{(y-1)} - 1)^2 \left( \xi_L^{(y-1)} - 1 \right)^2} \]  \hspace{1cm} (62)
The unknown \( \xi_L \) may be determined from Equation (62) by a numerical iterative technique. As before, \( \xi_L \) must exceed unity.

The heat flux at the fin base may be written as

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\[ q'' = \left[ C(\gamma T_0 - T_L) - \gamma T_m^4 (T_0 - T_L) \right] + [\gamma (T_L - T_m^4)]^{\frac{1}{2}} \] \hfill (63)

and the heat flux at the end of the fin is

\[ q''_L = \gamma (T_L - T_m^4) \] \hfill (64)

4. COMPUTER PROGRAMS

The solution of the equations for temperature distribution, root and outer end heat transfer rates, for all of the eight cases has been programmed for numerical computation on a digital computer. Then programs and their input and output format are given in the appendix.
<p>| Case | $\alpha_n$ | L | $\frac{d\theta}{dx}\bigg|<em>{\theta</em>{\text{max}}}$ | $\theta_{\text{max}}$ | C |
|------|------------|---|---------------------------------|-----------------|---|
| I    | 0          | 0 | 0                               | 0               | 0 |
| II   | 0          | Finite | 0                             | Unknown         | $-B\alpha_L^y$ |
| III  | 0          | Finite | $-\frac{2\alpha}{k_1}(\alpha a_{L1})^{y-1}$ | Unknown         | $-B\alpha_L^y(1 - \frac{2\alpha}{k_1}(\alpha a_{L1})^{y-2})$ |
| IVa  | 0          | Finite | Unknown                         | $\theta_L$      | $p_L^2 - B\alpha_L^y$ |
| IVb  | 0          | Finite | 0                              | Unknown         | $-B\alpha_L^y$ |
| IVc  | 0          | Finite | Unknown                         | $\theta_L, \theta_n$ | $-B\alpha_L^y$ |
| V    | $&gt;0$       | 0 | 0                               | $\theta_n$      | $-B\alpha_L^y(1 - \gamma)$ |
| VI   | $&gt;0$       | Finite | 0                             | Unknown         | $-B\alpha_L^y(\gamma_{L1} - \theta_{L1})$ |
| VIIa | $&gt;0$       | Finite | Unknown                         | $\theta_L &lt; \theta_n$ | $-B\alpha_L^y(\gamma_{L1} - \theta_{L1}) + p_L^2$ |
| VIIb | $&gt;0$       | Finite | Unknown                         | $\theta_L = \theta_n$ | $-B\alpha_L^y(1 - \gamma) + p_L^2$ |
| VIIc | $&gt;0$       | Finite | Unknown                         | $\theta_L &gt; \theta_n$ | $-B\alpha_L^y(\gamma_{L1} - \theta_{L1})$ |
| VIII | $&gt;0$       | Finite | $-\frac{2\alpha}{k_1}(\alpha a_{L1})^{y-1}(\gamma_{L1}^{y-1}-1)$ | $\theta_L$ | $-B\alpha_L^y(\gamma_{L1} - \theta_{L1}) + \frac{2\alpha}{k_1}(\alpha a_{L1})^{y-2}(\gamma_{L1}^{y-1} - 1)^2$ |</p>
<table>
<thead>
<tr>
<th>Case</th>
<th>Variable</th>
<th>( x )</th>
<th>( q_x^0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1</td>
<td>( \frac{2k_0 \Delta T}{c_p (3 \pi)^2} ) ( \frac{1}{2} ) ( \frac{\sigma-1}{\sigma+2} ) ( \frac{\sigma^2}{\sigma+2} ) ( \frac{\sigma-2}{\sigma+2} ) ( \frac{\sigma^2}{\sigma+2} )</td>
<td>( \left[ \frac{G(\gamma^0)}{L} \right] \frac{1}{2} )</td>
</tr>
<tr>
<td>II</td>
<td>2</td>
<td>( \frac{a_0^2}{a_0^2} ) ( \frac{1}{2} ) ( \frac{a_0^2}{a_0^2} ) ( \frac{a_0^2}{a_0^2} ) ( \frac{a_0^2}{a_0^2} ) ( \frac{a_0^2}{a_0^2} )</td>
<td>( \left[ \frac{G(\gamma^0 - \gamma^0_L)}{L} \right] \frac{1}{2} )</td>
</tr>
<tr>
<td>III</td>
<td>3</td>
<td>( \frac{1}{a_0^2 (\gamma-2)} ) ( \frac{1}{2} ) ( \frac{1}{a_0^2 (\gamma-2)} ) ( \frac{1}{a_0^2 (\gamma-2)} ) ( \frac{1}{a_0^2 (\gamma-2)} ) ( \frac{1}{a_0^2 (\gamma-2)} )</td>
<td>( \left[ \frac{G(\gamma^0 - \gamma^0_L)}{L} \right] \frac{1}{2} )</td>
</tr>
<tr>
<td>IVa</td>
<td>4</td>
<td>( \frac{1}{a_0^2 (\gamma-2)} ) ( \frac{1}{2} ) ( \frac{1}{a_0^2 (\gamma-2)} ) ( \frac{1}{a_0^2 (\gamma-2)} ) ( \frac{1}{a_0^2 (\gamma-2)} ) ( \frac{1}{a_0^2 (\gamma-2)} )</td>
<td>( \left[ \frac{G(\gamma^0 - \gamma^0_L)}{L} \right] \frac{1}{2} )</td>
</tr>
<tr>
<td>IVb</td>
<td>5</td>
<td>( \frac{1}{a_0^2 (\gamma-2)} ) ( \frac{1}{2} ) ( \frac{1}{a_0^2 (\gamma-2)} ) ( \frac{1}{a_0^2 (\gamma-2)} ) ( \frac{1}{a_0^2 (\gamma-2)} ) ( \frac{1}{a_0^2 (\gamma-2)} )</td>
<td>( \left[ \frac{G(\gamma^0 - \gamma^0_L)}{L} \right] \frac{1}{2} )</td>
</tr>
<tr>
<td>IVc</td>
<td>6</td>
<td>( \frac{1}{a_0^2 (\gamma-2)} ) ( \frac{1}{2} ) ( \frac{1}{a_0^2 (\gamma-2)} ) ( \frac{1}{a_0^2 (\gamma-2)} ) ( \frac{1}{a_0^2 (\gamma-2)} ) ( \frac{1}{a_0^2 (\gamma-2)} )</td>
<td>( \left[ \frac{G(\gamma^0 - \gamma^0_L)}{L} \right] \frac{1}{2} )</td>
</tr>
<tr>
<td>IVd</td>
<td>7</td>
<td>( \frac{1}{a_0^2 (\gamma-2)} ) ( \frac{1}{2} ) ( \frac{1}{a_0^2 (\gamma-2)} ) ( \frac{1}{a_0^2 (\gamma-2)} ) ( \frac{1}{a_0^2 (\gamma-2)} ) ( \frac{1}{a_0^2 (\gamma-2)} )</td>
<td>( \left[ \frac{G(\gamma^0 - \gamma^0_L)}{L} \right] \frac{1}{2} )</td>
</tr>
</tbody>
</table>

**TABLE III**

Temperature Distribution and Heat Transfer Equations for the Radiating Fin
<table>
<thead>
<tr>
<th>Case</th>
<th>Variable</th>
<th>$x$</th>
<th>$q''_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>$\xi = \frac{\theta}{\theta_m}$</td>
<td>$\left[\frac{1}{B_0 (\gamma-2)}\right]^{1/2} \int_{\xi}^{\xi_o} \frac{d\xi}{[\xi - 1 - \gamma (\xi - 1)]^{1/2}}$</td>
<td>$\left(\gamma_0^{\beta} - \gamma_0^{\gamma} \tau_0^{\beta} + (\gamma-1) \tau_0^{\beta} \right)^{1/2}$</td>
</tr>
<tr>
<td>VI</td>
<td>$\xi = \frac{\theta}{\theta_m}$</td>
<td>$\left[\frac{1}{B_0 (\gamma-2)}\right]^{1/2} \int_{\xi}^{\xi_o} \frac{d\xi}{[\xi - 1 - \gamma (\xi - 1)]^{1/2}}$</td>
<td>$\left(\gamma_0^{\beta} - \gamma_0^{\gamma} \tau_0^{\beta} (\gamma_0^{\alpha} - \tau_0^{\alpha}) \right)^{1/2}$</td>
</tr>
<tr>
<td>VIIa</td>
<td>$\xi = \frac{\theta}{\theta_m}$</td>
<td>$\left[\frac{1}{B_0 (\gamma-2)}\right]^{1/2} \int_{\xi}^{\xi_o} \frac{d\xi}{[\xi - 1 - \gamma (\xi - 1)]^{1/2}}$</td>
<td>$\left(\gamma_0^{\beta} - \gamma_0^{\gamma} \tau_0^{\beta} (\gamma_0^{\alpha} - \tau_0^{\alpha}) \right)^{1/2}$</td>
</tr>
<tr>
<td>VIIb</td>
<td>$\xi = \frac{\theta}{\theta_m}$</td>
<td>$\left[\frac{1}{B_0 (\gamma-2)}\right]^{1/2} \int_{\xi}^{\xi_o} \frac{d\xi}{[\xi - 1 - \gamma (\xi - 1)]^{1/2}}$</td>
<td>$\left(\gamma_0^{\beta} - \gamma_0^{\gamma} \tau_0^{\beta} (\gamma_0^{\alpha} - \tau_0^{\alpha}) \right)^{1/2}$</td>
</tr>
<tr>
<td>VIIc</td>
<td>$\xi = \frac{\theta}{\theta_m}$</td>
<td>$\left[\frac{1}{B_0 (\gamma-2)}\right]^{1/2} \int_{\xi}^{\xi_o} \frac{d\xi}{[\xi - 1 - \gamma (\xi - 1)]^{1/2}}$</td>
<td>$\left(\gamma_0^{\beta} - \gamma_0^{\gamma} \tau_0^{\beta} (\gamma_0^{\alpha} - \tau_0^{\alpha}) \right)^{1/2}$</td>
</tr>
</tbody>
</table>
### Table III (continued)

<table>
<thead>
<tr>
<th>Case</th>
<th>Variable</th>
<th>( x )</th>
<th>( \xi_{0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIII</td>
<td>( \xi = \frac{\theta}{\theta_{0}} ) ( 1 + \frac{1}{B_{0}(\gamma-2)} ) ( F_{0} )</td>
<td>( \frac{d\xi}{\xi} \left{ 1 - \frac{\xi}{\xi_{L}} - \frac{\gamma}{\xi - \xi_{L}} - 1 \right} \frac{d\xi}{\xi_{L}} ) ( (\gamma-2) \left( \frac{\xi_{L}}{\xi_{0}} - 1 \right)^{2} )</td>
<td>( \left[ g(\xi_{0} - \xi_{L}) - g(\xi_{0} - \xi_{L}) \right] + \left[ \cos(\xi_{L} - \xi_{L}) \right] \frac{1}{2} )</td>
</tr>
</tbody>
</table>
SECTION IV

LIMITATIONS ON THERMAL SCALE MODELING

1. DIMENSIONAL LIMITATIONS

The thermal modeling parameters listed as the method 1 parameters of Table I of Section II are exact. From a theoretical standpoint there are no limitations on the size of the model used to predict prototype parameters. From a practical standpoint however, there are limitations to the accuracy to which a model may be constructed.

To obtain information on the magnitude of prototype prediction errors which may be caused by model dimensional errors, a detailed study of the temperatures and heat fluxes in a radiating cylindrical fin will be made. The fin has a fixed root temperature and radiates to black surroundings from both the cylindrical surface and the outer end area. This is case VIII of Section III. Although the equations of Section III are derived for a rectangular fin, it may be shown that they apply to a cylindrical fin if one-fourth of the fin diameter is used in place of the rectangular fin half-thickness.

The fin will be considered to be made of 2024 annealed aluminum, for which Lucks and Deen (5) have reported values of thermal conductivity as a function of temperature. The reported conductivity of this material at 528°F is 103 Btu/hr-ft-°R and an analysis of the data at other temperatures indicates that the equation

$$k = 29.37(T^{0.1980})$$

will predict the conductivity within 2% over the temperature range of 210
to 860°F. See Appendix B for complete conductivity data.

The exact model of the prototype fin is to be one-fourth size and not distorted, that is the length and diameter are to be scaled in the same ratio. Prototype and model dimensions, temperatures, heat fluxes and other properties are given in Table IV.

Using the model data and the method 1 parameters of Table 1 it is possible to calculate prototype temperatures and fluxes. These results are given in Table V. By inspection it is seen that the prototype quantities calculated from the model results are negligibly different from those directly calculated for the prototype.

To demonstrate the prototype prediction errors caused by errors in the dimensions of the model, calculations of temperatures and fluxes were made for models which had length and diameter errors of 1, 2, 5, 8 and 10 percent, both above and below the exact dimension. The prototype prediction results are shown in Figures 3 and 4. These figures show that zero prototype prediction error occurs with ± 1% error in model diameter and ±1% error in model length. If the modeling laws are exact this should not occur. The solutions were obtained by numerical methods, however, and this apparent discrepancy is caused by the numerical integration scheme used in obtaining the solution.

These figures show that a ±1% error in either of the model dimension will result in less than ±4% error in the fin tip temperature and less than ±5% errors in the fin root and tip heat transfer rates. The exact model is 0.500 inches in diameter and 6 inches in length so this percentage

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### TABLE IV
Prototype and Exact Fin Data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prototype</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Root temperature, ( T_0 ) (°F)</td>
<td>530.0</td>
<td>869.2</td>
</tr>
<tr>
<td>Fin length, ( L ) (ft)</td>
<td>2.000</td>
<td>0.500</td>
</tr>
<tr>
<td>Fin diameter, ( D ) (in)</td>
<td>2.000</td>
<td>0.500</td>
</tr>
<tr>
<td>Emittance, ( e )</td>
<td>0.950</td>
<td>0.950</td>
</tr>
<tr>
<td>Exponent, ( n )</td>
<td>0.1980</td>
<td>0.1980</td>
</tr>
<tr>
<td>Constant, ( k_1 )</td>
<td>29.37</td>
<td>29.37</td>
</tr>
<tr>
<td>Surrounds Temp., ( T_e ) (°F)</td>
<td>160.0</td>
<td>160.0</td>
</tr>
</tbody>
</table>

**Calculated Data**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prototype</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer end temp., ( T_L ) (°F)</td>
<td>483.5</td>
<td>792.3</td>
</tr>
<tr>
<td>Outer end heat transfer ( q_L ) (Btu/hr)</td>
<td>1.941</td>
<td>1.875</td>
</tr>
<tr>
<td>Root end heat transfer ( q_0 ) (Btu/hr)</td>
<td>107.0</td>
<td>48.73</td>
</tr>
</tbody>
</table>

### TABLE V
Calculated Data for Prototype Fin

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prototype</th>
<th>Calculated Prototype</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_0 ) (°F)</td>
<td>530.0</td>
<td>530.0 (°F)</td>
<td>0.0 (%)</td>
</tr>
<tr>
<td>( T_L ) (°F)</td>
<td>483.5</td>
<td>483.2 (°F)</td>
<td>0.3 (%)</td>
</tr>
<tr>
<td>( q_L ) (Btu/hr)</td>
<td>1.941</td>
<td>1.935 (Btu/hr)</td>
<td>-0.3 (%)</td>
</tr>
<tr>
<td>( q_0 ) (Btu/hr)</td>
<td>107.0</td>
<td>107.7 (Btu/hr)</td>
<td>+0.7 (%)</td>
</tr>
</tbody>
</table>

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Figure 3. Prototype prediction errors caused by model dimension errors (exact model).
FIGURE 4. PROTOTYPE PREDICTION ERRORS CAUSED BY MODEL DIMENSION ERRORS (EXACT MODEL)
error amounts to 0.025 inches on the diameter and 0.30 inches on the length. From a manufacturing tolerance standpoint, these are very large errors, and the probability of their occurrence should be very small.

Combinations of length and diameter errors could occur and could either tend to null out the effects of each other if they were in opposite directions or reinforce each other if in the same direction.

2. DISTORTED MODEL ERRORS

To simplify the thermal modeling procedure, it would be convenient to have the model and prototype operate at the same temperature at homologous points. For models made of the same material as their prototype, this is possible only by distorting the model dimension, that is, not scaling by the same fraction in all directions. This procedure is not exact however, and the resulting prototype predictions are in error, the amount depending upon the degree of distortion of the model. The method 2 parameters of Table I were derived for the specific case under study here; a solid radiating cylinder. To obtain temperature equality between model and prototype at homologous locations the radius is scaled in proportion to the square of the length. To determine the amount of prototype prediction errors caused by model distortion, calculations were made for the distorted model-prototype system described in Table VI. The prototype is the same as used in the previous case of exact modeling. The model length is one-fourth of the prototype length and the model diameter is one-sixteenth of the prototype diameter. This is also a case VIII problem as previously described.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prototype</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_0$ (ft)</td>
<td>530.0</td>
<td>530.0</td>
</tr>
<tr>
<td>L (ft.)</td>
<td>2.000</td>
<td>0.500</td>
</tr>
<tr>
<td>D (in.)</td>
<td>2.000</td>
<td>0.125</td>
</tr>
<tr>
<td>$e$</td>
<td>0.950</td>
<td>0.950</td>
</tr>
<tr>
<td>a</td>
<td>0.1980</td>
<td>0.1980</td>
</tr>
<tr>
<td>$k_1$</td>
<td>29.37</td>
<td>29.37</td>
</tr>
<tr>
<td>$r_\infty$ (ft)</td>
<td>160.0</td>
<td>160.0</td>
</tr>
</tbody>
</table>

### Calculated Data

| $T_L$ (k)   | 483.5    | 484.3  | +0.8 (%) |
| $q_L$ (Btu/hr) | 1.941   | 0.489  | -74.8 (%) |
| $q_o$ (Btu/hr)   | 107.0   | 106.2  | -0.75 (%) |
Figures 5 and 6 show prototype prediction errors for two of the fin parameters, the temperature at the fin tip and the heat transfer rate at the fin root. These two predictions are in error by approximately the same amounts as those for the exact model.

The heat transfer rate at the tip of the fin is approximately -75% in error, as shown in Table VI. The variation with model dimension errors is not so significant as the absolute error so the data are not shown graphically.

The cause of the error in the heat transfer at the fin tip is the distortion in the model. When the fin diameter is scaled as the square of the length, the radiation area at the tip of the fin is scaled by the fourth power of the length, resulting in the heat transfer rate at the fin tip also being scaled as the fourth power of the length since the temperature and emissive of model and prototype are equal. The cylindrical surface area of the fin, however, is scaled to the third power of the length. This results in the cylindrical surface radiation heat transfer being scaled to the third power of the length.

The conduction heat transfer rate is scaled according to the ratio, cross-sectional area/length since the temperatures are the same at analogous locations, making the thermal conductivity the same in model and prototype. The ratio, cross-sectional area/length reduces to a conduction heat transfer scaling of length to the third power, the same as the cylindrical surface area heat transfer but different than the radiating tip heat transfer.
FIGURE 5. Prototype prediction errors caused by model dimension errors (disturbed model)
Figure 6. Prototype prediction errors caused by model dimension errors (distorted model).
The effect of the radiating tip area is very small at the root of the fin, thus the error in predicted prototype heat transfer at the fin root is also very small. As the tip of the fin is approached, however, the error in the prediction of conduction heat transfer will increase to a maximum of -75% error at the tip of the fin.

For the prediction of fin temperature, the distorted model technique is valuable and sufficiently accurate. Large errors in heat transfer rates can occur, however, which is very undesirable.

3. EFFECTS OF CONDUCTIVITY VARIATION WITH TEMPERATURE

For this situation we will consider that the experimenter will assume that both model and prototype have thermal conductivities which do not vary with temperature, while in fact both the model and the prototype do have thermal conductivities which vary with temperature. We will investigate the prediction errors the experimenter would make for the same prototype used in the previous two cases. For this situation the prediction would be made on the basis of the method 3 parameters of Table I, assuming a one-fourth scale model. The model root temperature is calculated from dimensional ratios to be 84.3°F, based on a prototype root temperature of 530°F. Table VII gives a comparison of the actual prototype and model data and the errors incurred by the assumption of constant thermal conductivity.

Generally speaking, these errors are not large. However, the magnitude of such errors will increase with the degree of dependence of thermal conductivity on temperature. Table VIII shows the same type data for a
### TABLE VII
Prototype and Model Data for Constant Thermal Conductivity

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prototype</th>
<th>Model</th>
<th>Predicted Prototype</th>
<th>Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_o$ (°F)</td>
<td>530.0</td>
<td>841.3</td>
<td>530.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$T_L$ (°F)</td>
<td>483.5</td>
<td>772.1</td>
<td>486.4</td>
<td>+2.9</td>
</tr>
<tr>
<td>$q_o$ (Btu/hr)</td>
<td>107.0</td>
<td>44.5</td>
<td>109.5</td>
<td>+2.3</td>
</tr>
<tr>
<td>$q_L$ (Btu/hr)</td>
<td>1.941</td>
<td>0.789</td>
<td>1.988</td>
<td>+2.4</td>
</tr>
</tbody>
</table>

### TABLE VIII
Prototype and Model Data for Constant Thermal Conductivity

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prototype</th>
<th>Model</th>
<th>Predicted Prototype</th>
<th>Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_o$ (°F)</td>
<td>530.0</td>
<td>841.3</td>
<td>530.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$T_L$ (°F)</td>
<td>483.5</td>
<td>779.4</td>
<td>491.0</td>
<td>+7.5</td>
</tr>
<tr>
<td>$q_o$ (Btu/hr)</td>
<td>107.1</td>
<td>44.5</td>
<td>98.5</td>
<td>-8.0</td>
</tr>
<tr>
<td>$q_L$ (Btu/hr)</td>
<td>1.941</td>
<td>0.819</td>
<td>1.812</td>
<td>-6.7</td>
</tr>
</tbody>
</table>

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prototype-model system made of a fictional material which has a thermal conductivity related by the following equation

$$k = 4.482 \times 10^{-0.5}$$  \hspace{1cm} (66)

This material has a conductivity of 103 Btu/hr-ft-°R at 320°F, the same as the 2024 aluminum previously considered, but a larger variation of conductivity with temperature, and the prediction errors are significantly larger.

4. EFFECTS OF THE RADIATION ENVIRONMENT TEMPERATURE

In the previous cases we have considered the prototype and models to be situated in a radiation environment of 260°F. To determine the effects of the radiation environment we will consider again the same prototype and alter the temperature of the surroundings from 4°F to 350°F and calculate the errors in fin end temperatures and root end outer end heat transfers. The errors will be based on the differences from the 4°F values.

Figure 7 shows these results. With an environment temperature of 360°F, the errors in the heat transfer rates are less than 2 percent and the fin tip temperature is in error by 1.5°F. These are negligible errors in most cases and generally speaking would not be large enough to warrant using any cooling means but liquid nitrogen to attempt to obtain a lower environmental temperature.

5. MODELING WITH LOW PROTOTYPE TEMPERATURES

As prototype operating temperatures are decreased, the heat transfer
\[ \Delta \text{HEAT TRANSFER ERROR AT FIN TIP (\%)} \]
\[ \square \text{HEAT TRANSFER ERROR AT FIN ROOT (\%)} \]
\[ \circ \text{TEMPERATURE ERROR AT FIN TIP (R)} \]

**Figure 7. Surroundings Temperature Effect on Prototype Prediction Errors**
quantities decrease and there is a greater possibility of prediction errors from model studies. To obtain some idea of the magnitude of the errors involved in such a situation, a series of computer calculated temperature distributions and heat transfer rates were obtained for a prototype fin with root temperatures ranging from 200°F to 530°F were obtained. The prototype fin was the same size and material as that considered previously. It was a 2 inch diameter solid cylinder, 2 ft. long, and made of 2024 aluminum. The model was one-fourth scale before, and made of the same material. The corresponding model root temperatures varied from 328.0°F to 869.2°F. For both prototype and model the radiation environment temperature was 4 K.

The results are shown in Figure 8. The errors in predicted prototype temperature and Fin tip heat transfer rate decreased as the Fin root temperature decreased, reaching essentially zero error at 200°F. The percentage errors in predicted prototype root heat transfer rate increased as the prototype temperatures were decreased, from 11 at 530°F to 7.4% at 200°F.

6. COMPLEX RADIATIVE/CONDUCTIVE INTERCHANGES

The thermal scale modeling of structures which have multiple heat transfer paths utilizing different modes of heat transfer is not a difficult process if exact scale modeling is used. However, if it is necessary to distort some dimensions in the model, the prediction of prototype parameters from model data becomes increasingly difficult as the degree of distortion increases.

One of the objectives of this research was to investigate the effects of model distortion when complex radiative/conductive interchanges were in-
\[ \triangle \text{HEAT TRANSFER ERROR AT FIN TIP (\%)} \]

\[ \square \text{HEAT TRANSFER ERROR AT FIN ROOT (\%)} \]

\[ \bigcirc \text{TEMPERATURE ERROR AT FIN TIP (\%)} \]

**FIGURE 8.** PROTOTYPE TEMPERATURE EFFECTS ON PROTOTYPE PREDICTION ERRORS
volved. This problem has been discussed by several authors, notably Barcus in 1966(12). The scheme which Barcus proposes is a "computer-assisted" technique using distorted models. The model is distorted as necessary and several tests at different conditions are made on the model. These results are then utilized as input data to a computer which solves many simultaneous equations to calculate prototype temperatures and heat transfers for a single prototype condition. Several model tests are necessary for each prototype prediction to account for the distortion in the model.

Although considerable effort has been expended during the term of this contract, it has not been possible to accomplish the objective of determining the effects of complex heat exchange paths when using distorted models.
SECTION V

SUMMARY OF RESULTS

1. Solutions were presented for the steady-state temperature distribution and heat transfer rates in a radiating fin having temperature dependent thermal conductivity. There were solutions for eight different cases of boundary conditions applied to the fin. The solutions were exact but numerical means were used for integration purposes so computer programs, in Fortran language, for obtaining the solutions were also presented. This analysis, and the computer programs, were used to determine the limitations on accuracy in thermal scale modeling.

2. It has not been possible to obtain a generalized method of error prediction for thermal modeling. It was, however, possible to determine the magnitude of prototype prediction errors for the case of steady-state radiating fins. Through a study of the prediction errors of these fin systems it was possible to obtain a better insight into the magnitude of errors involved in the use of thermal modeling techniques.

3. It was determined that, for an exact thermal model, model length or diameter errors of ±5% caused less than 4% error at the tip of a radiating end fin and less than ±5% error in the fin root and tip heat transfer rates.

4. For a severely distorted model, ±5% errors of model diameter or length caused approximately the same magnitude error in fin tip temperature and root heat transfer as in the exact model. The error in heat transfer rate increased along the length of the fin, however, to -75% at the fin tip. The cause of this prediction error was the distortion in the model.
5. For fin systems which were constructed of material having temperature dependent thermal conductivity, but for which the experimenter assumed constant thermal conductivity, the prediction errors increased with increasing dependence of thermal conductivity on temperature. Examples of prediction errors were given for a fin made of aluminum and another (fictional) material having greater changes of conductivity with temperature.

6. Liquid nitrogen, at 160°F, is often used in space simulators to simulate the low temperature of outer space. Depending upon the circumstances, this simulation may not be adequate. To determine the approximate magnitude of the errors involved in such a situation, a determination of fin temperatures and heat transfer rates was made for environmental temperatures between 50°F and 350°F. These were compared with values calculated for an environmental temperature of 4°F. The errors increase in magnitude with the environmental temperature, of course, and typical results with an environmental temperature of 160°F show that the fin tip temperature error was 1.3°F and the heat transfer rates at the fin root and tip were in error by less than 2%.

7. For a prototype fin with a low operating temperature (fin root temperature from 530°F down to 200°F) the errors in predicted prototype tip temperature and heat transfer rate decreased as the fin root temperature decreased and were always below 1°F and 12 respectively. The errors in root heat transfer rate increased from 1% at 530°F to 7% at 200°F.

8. Attempts were made to determine the effects of distorted models when complex radiative/conductive heat exchange paths were involved but this objective was not accomplished.
REFERENCES


APPENDIX A

COMPUTER PROGRAMS

The eight cases of boundary conditions which apply to the radiating fin are, for convenience, presented as eight separate computer programs. The input data for all programs uses exactly the same format specifications so the separate program could easily be incorporated as subroutines in one large program if desired. The input data format and definition of variables is presented in Table IX and the program listings are presented in Table X through Table XVII.

The output data has virtually the same form for all programs. Typical output data forms for each of the eight programs are shown in Table XVIII through Table XXV. All input data is printed as part of the output as well as calculated data, including fin distances as a function of temperature, minimum temperatures if they occur, and heat fluxes at various locations on the fin, such as at the root and at the outer end of the fin.
Input Data Format

Radiating Fin Computer Programs

Fortran IV Language

A. The first input card states the number of data sets (2 cards per set) which follow. The field is 10.

B. Input to each program consists of two cards, the first having eight F 10.4 fields, the second having two I 10 fields, two F 10.4 fields and one I 10 field.

C. Input data names for the first card are:

TO: Fin root temperature (R)
TINF: Surroundings temperature (R)
DEL: Fin half thickness, if rectangular or one-fourth of fin diameter if circular (FT)
E: Fin surface emittance
A: Exponent in the equation k = k₀ T⁰
XED: Constant k₀ in the previous equation. K must have units of (Btu/hr-ft.R).
XL: Fin length (FT)
IT: Fin temperature at outer end. Leave blank if not applicable. (R)

D. Input data names for the second card are:

IXNO: Temperature intervals at which fin distances from the root are calculated (R)
N: The number of points at which the subroutine Riemann estimates the second derivative of the function to decide on new sub interval lengths (N=3). For most functions N = 10 is sufficient.
EPS: The absolute limit of error for the Riemann subroutine integration. Because of roundoff 1 x 10⁻⁶ is the recommended lower limit of EPS.
TABLE II (cont)

ACC:  The accuracy to which specified and calculated fin length must agree before iteration is ended. $1 \times 10^{-3}$ is usually sufficient (P2).

IYPE:  Dummy data name. Not used in the program.
CASE I COMPUTER PROGRAM

C**********SOLUTION TO CASE 1 PROBLEM**********
1 FORMAT(BF10.4)
2 FORMAT(1E10.2,1I10)
101 FORMAT(H11,10X,"CASE 1 PROBLEM"/)
112 FORMAT(20X,"HEAT FLUX AT ROOT =",F10.2," Btu/HR-SQFT")
114 FORMAT(20X,4X,FL="F0.3",4H FT,3X,4H HT = "F0.2",4H DEG.R")
121 FORMAT(11X,"INFINITE LENGTH FIN")
122 FORMAT(11X,"INFINITY = "F8.2"," R")
123 FORMAT(11X,"EMITTANCE = "F6.3)
124 FORMAT(11X,"FIN HALF THICKNESS =",F12.7," FT")
125 FORMAT(11X,"EXPONENT = "F7.4")
126 FORMAT(11X,"ROOT TEMP. = "F8.2"," R")
127 FORMAT(11X,"CONDUCTIVITY = "F7.2)
READ(1,2) II TIMES
READ(1,1) TO,TINF,DEL,E,A,XXO,XL,TL
WRITE(3,101)
WRITE(3,121)
WRITE(3,126)TD
WRITE(3,122)TNT
WRITE(3,123)E
WRITE(3,124)DEL
WRITE(3,127)XXO
WRITE(3,1251A)
SIGMA=0.1714E-08
GAMA=(A+5.1)/(A+1.)
B=SIGMA*(A+1.)**GAMA/XXO*DEL*(A+5.1)**2.
G2=SIGMA*XXO/(DEL*(A+5.1))
WRITE(3,114)TINF,T0
T0=1/FIX((T0*100)/100.)
T0=100*MTO
XNO = FLOAT(IXXO)
L = 1
343 IF(T0.GT.(T0-100)) GO TO 343
IF(T0.LT.(TINF-10)) GO TO 370
X = SQRT((X2*(A+1.))**GAMA)/(B*(3.-A)**2.)
Y = -1./SQRT((T0**((3.-A)) + 1./SQRT(T0**((3.-A))))
X = X*Y
WRITE(3,114)X,T
GO TO 343
370 CONTINUE
QO = SQRT(T0*(T0*(A+5.1)))
WRITE(3,112)QO
1000 CONTINUE
STOP
END

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**TABLE XI**

CASE II COMPUTER PROGRAM

C**********SOLUTION TO CASE 2 PROBLEM**********

\[ \text{Theta}(x, y) = (y*(x + 1.))/(x + 1.) \]

1 FORMAT(F8.10,4)

2 FORMAT(215I3,2F10.4,1I0)

112 FORMAT(/4,20X,1H18 FLUX AT ROOT = ','F10.2,1 BTU/HR-SQ FT')

114 FORMAT(20X,4H4 XFB,3+H FT,2,3X,4H = ,F8.2,1 DEG, R')

122 FORMAT(11X,* T INFINITY = ,F8.2,1 R')

123 FORMAT(11X,*EMITTANCE = ,F6.3)

124 FORMAT(11X,*FIN HALF THICKNESS = ,F12.7,1 FT')

125 FORMAT(11X,*EXponent = ',F7.4/)

126 FORMAT(11X,*ROOT TEMP. = ,F8.2,1 R')

127 FORMAT(11X,*CONDUCTIVITY = ',F7.2)

201 FORMAT(11H1,10X,'CASE 2 PROBLEM'/)

206 FORMAT(11X,*FIN LENGTH = ',F8.3,1 FT')

207 FORMAT(11X,*TEMPERATURE AT OUTER END = ',F8.2,1 DEG. R'/)

221 FORMAT(11X,*FINITE LENGTH FIN WITH INSULATED END')

1C01 FORMAT(/4,11X,*N = ',I4)

1C02 FORMAT(11X,*EPS = ',F9.7)

1C03 FORMAT(11X,*ACC = ',F9.7)

READ(1,2) ITIMES

4 DO 1C00 IDA=1,ITIMES

READ(1,1) TO,TINF,DEL,Ep,A,XKD,XXL,TL

READ(1,2) IXNO,N,EPS,ACC,ITYPE

WRITE(3,201)

WRITE(3,221)

WRITE(3,206)XL

WRITE(3,126)O

WRITE(3,122)TINF

WRITE(3,123)E

WRITE(3,124)DEL

WRITE(3,127)XKD

WRITE(3,1251A)

SIGMA=0.1714E-08

GAMA=(A+5.7)/A**GAMA/(XKD*DEL*1A+5.1)**2.

G=2.*SIGMA*KKD/(DEL*(A+5.1))

A1 = 1.

A2 = 1.01

A3 = 0.

A4 = 0.

A5 = -1.

A6 = GAMA

R = RIEMANA1,A2,A3,A4,A5,A6,N,EPS)

A1 = A2

X = SQRT(B*THETA(A,TO)**(GAMA-2.1))

TL = 0

W = .5*TO

251 TL = TL + W

A2 = THEtA(A,TO)/THEtA(A,TL)

IF(A2.LT.1.015) R = 0

IF(A2.LT.1.015) A1 = 1

219 Y = R * RIEMANA1,A2,A3,A4,A5,A6,N,EPS)

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Y = Y*SQRAT(A2**GAMMA-2,1)/X
XL1 = Y
IF(ABS(XL-Y1.1,.1,ACC) GO TO 6
IF(Y-XL) 220,6,222
220 TL = TL - W
W = .5*W
GO TO 251
251 W = .5*W
GO TO 251
6 WRITE(3,207) TL
WRITE(3,114) A1,TL
XX = 1.01
TT0 = IFIX(100*100)
TT0 = 100*TT0
XNO = FLOAT((XNO)
L = 1
A2 = THEATA(A2,TT0)/THEATA(TL,TL)
Z = 1./SQRAT(2*THEATA(A2,TL)**(GAMMA-2,1))
343 T = TT0*XNO
L = L + 1
IF(TT.TG.(TT-100) GO TO 343
IF(TT.LT.100) GO TO 370
A1 = THEATA(A2,TL)/THEATA(A2,TL)
Y1 = 0
IF(A1.LT.1,005) GO TO 252
Y1 = RIEAN(A1,XX,XA,XA,XA,XA,AE,AE,AE,AE,A,AE)
A1 = 1X
252 Y = TL + RIEAN(A1,A2,A3,A4,A5,A6,AE,AE)
Z = Z*Y
205 WRITE(3,114) X,Y
GO TO 343
370 CONTINUE
WRITE(3,114) X1,TL
GO = SQRAT(G**100*(A+5.1)-TL**2(A+5.1))
WRITE(3,112) GO
WRITE(3,2001) N
WRITE(3,1002) EPS
WRITE(3,1003) ACC
100 CONTINUE
STOP END
FUNCTION RIEAN (A1,A2,A3,A4,A5,A6,AE,AE)
DIMENSION A31,F3(1)
Y = RIEAN(A1,A2,A3,A4,A5,A6,A,X) = 1./SQRAT((X**A6-A3**A6-A4*A6*(X-A3)+A5))
100 DIV=E/(A2-A1)/FLOAT(N)
101 DO 103 L=1,2
102 A(L)=A1+FLOAT(L-2)/2.0)*DIVA
103 F(L) = VALUE(A1,A2,A3,A4,A5,A6,A(L))
104 END=A1
105 RIEAN=0.0
199 DO 233 I=1,N
1591 RESULT=0.0

53
200 ERR = (F(1)-2.0*F(2)+F(11))*DIVA/24.0*FLOAT(N)
201 ERR = ABS(ERR/EPS)
202 IF (ERR<1.0) 218, 218, 203
203 K = SQRT(ERR)+1.0
204 L = K/2
205 K = 2*L+1
206 DIVR = DIVA/FLOAT(K)
207 DIVR2 = DIVR/2.0
208 JD 211 J = 1, L
209 ADD = VALUE(A1, A2, A3, A4, A5, A6, END+DIVR2)*DIVR
210 RESULT = RESULT+ADD
211 END = END+DIVR
212 END = END+DIVR
213 DD 216 J = 1, L
214 ADD = VALUE(A1, A2, A3, A4, A5, A6, END+DIVR2)*DIVR
215 RESULT = RESULT+ADD
216 END = END+DIVR
217 GO TO 220
218 END = END+DIVA
219 DIVR = DIVA
220 IF (I-1) 222, 222, 221
221 IF (I-(N-1)) 224, 224, 229, 229
222 RESULT = RESULT+F(11)*DIVR
223 GO TO 231
224 RESULT = RESULT+F(2)+DIVR
225 F(11) = F(2)
226 F(2) = F(3)
227 F(11) = VALUE(A1, A2, A3, A4, A5, A6, END+13./2.)*DIVA
228 GO TO 233
229 IF (I-N) 230, 230, 232, 232
230 RESULT = RESULT+F(2)*DIVR
231 GO TO 233
232 RESULT = RESULT+F(3)*DIVR
233 RIEMAN = RIEMAN+RESULT
234 RETURN
ENG
C**********SOLUTION TO CASE 3 PROBLEM**********

\[
\theta(x,y) = \frac{Y^2(x + 1)}{1 + 1}
\]

1 FORMAT(6F10.4)
2 FORMAT(21050Z, F10.4, 110)
112 FORMAT(2X, 2O, 'HEAT FLUX AT R O O T = ', F10.2, ' BTU/HR-SQFT')
114 FORMAT(2X, 4X, 'F8.3,4H FT.3X,4HT = ', F8.2, ' DEG.R')
122 FORMAT(11X, 'INFINITY = ', F8.2, ' R')
123 FORMAT(11X, 'EMITTANCE = ', F6.3)
124 FORMAT(11X, 'FIN HALF THICKNESS = ', F12.7, ' F1')
125 FORMAT(11X, 'X,CONDUCTIVITY = ', F7.2, ' F1')
207 FORMAT(11X, 'TEMPERATURE AT OUTER END = ', F8.2, ' DEG. R')
209 FORMAT(11X, 'FIN LENGTH = ', F8.2, ' F1')
207 FORMAT(11X, 'FLUX AT OUTER END = ', F10.2, ' BTU/HR-SQFT')
1001 FORMAT(1X, 11X, 'N = ', I4)
1002 FORMAT(11X, 'EPS = ', F9.7)
1003 FORMAT(11X, 'ACC = ', F9.7)

READ(1,2) TIMES
DO 100 IDA = 1, TIMES
READ(1,3) T0, TINF, DEL, E, A, XK0, X1, TL
READ(1,2) INO, N, EPS, ACC, ITYPE
WRITE(3,301)
WRITE(3,221)
WRITE(3,226) X1
WRITE(3,229) TL
WRITE(3,122) TINF
WRITE(3,123) E
WRITE(3,124) DEL
WRITE(3,127) XK0
WRITE(3,125) A
SIGMA = 0.1714E-08
GAMA = (A + 5.0) / (A + 1.0)
B = SIGMA * (A + 1.0) * GAMA / (XK0 * DEL * (A + 5.1))
G = 2.0 * SIGMA * XK0 / (DEL * (A + 5.1))
A3 = 0.0
A4 = 0.0
A5 = -1.0
A6 = GAMA
W = 0.09
TL = 0.0
R = 1.01
I = 1.005
303 TL = TL + W
TAU = (1.0 - E * SIGMA * DEL * (A + 5) * TL ** (3 - A) / (2 * XK0)) ** 0.5
A2 = THETA(A, TL) / (THETA(A, TL) * TAU)
X = 1.0 * SQRT((THETA(A, TL) / TAU) * (GAMA - 2.0))
A1 = 1.0 / TAU
Y1 = 0
IF (A1 > 0.2) GO TO 304

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TABLE XII (cont)

Y1 = X*Riemann(A1, R, A3, A4, A5, A6, N, EPS)
A1 = R
304 Y2 = X*Riemann(A1, A2, A3, A4, A5, A6, N, EPS)
Y = Y1 + Y2
Y5 = Y
IF(ABS(XL-Y1), LT, ACC) GO TO 340
IF(XL-Y1) 306, 340, 307
307 TL = TL - W
W = .5*W
GO TO 303
306 W = .5*W
GO TO 303
340 WRITE(3, 2071) TL
WRITE(3, 114) A3, T0
IT0 = INT((T0+100.)/100.)
T0 = 100*IT0
END = FLOAT(IT0)
L = 1
341 IF(A3, GT, (T0-10.)) GO TO 343
IF(TL, LT, (T0+10.)) GO TO 370
A1 = THETA(A1, T)/THETA(A1, TL)*TAU
Y1 = 0
IF(A1, GT, 2) GO TO 341
Y1 = X*Riemann(A1, R, A2, A3, A4, A5, A6, N, EPS)
A1 = R
341 Y2 = X*Riemann(A1, A2, A3, A4, A5, A6, N, EPS)
Y = Y1 + Y2
WRITE(3, 114) Y, T
GO TO 343
370 WRITE(3, 114) Y5, TL
TAU = 1.-F*SIGMA*DEL*(A+5.)*TL**3/(3.-A)/(2.*K0)
G0 = SQRT(G*(A+5.1)-TL*(A+5.1)*TAU))
W = .5*SIGMA*TL**4
WRITE(3, 112) G0
WRITE(3, 413) G0
WRITE(3, 1001) N
WRITE(3, 1002) EPS
WRITE(3, 1003) ACC
1000 CONTINUE
STOP
END

FUNCTION Riemann (A1, A2, A3, A4, A5, A6, N, EPS)
DIMENSION A1(3), F(3)
VALUE(A1, A2, A3, A4, A5, A6, X)=1./SQRT(X**2-A3**2-A6**2*(X-A3)+A5**2)
100 DIVA=(A2-A1)/FLOAT(N)
101 DO 103 I=1,3
102 A(I)=A1+(FLOAT(I-1)/2.)*DIVA
103 F(I) = VALUE(A1, A2, A3, A4, A5, A6, A(I))
104 END
105 Riemann=0.0

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199 DD 233 I=1,N
1991 RESULT=0.0
200 ERROR= ((F(13)-2.0*F(2)+F(11))*DIVA1/24.0)*FLOAT(N)
201 ERR=ABS(ERROR/EP5)
202 IF (ERRK<1.0) 218, 218, 203
203 K=SQR(RESULT+1.0)
204 L=K/2
205 K=2*KE+1
206 DIVR=DIVA/FL0AT(K)
207 DIVR2=DIVR/2.0
208 DD 211 J=1,L
209 ADD = VALUE(A1,A2,A3,A4,A5,A6,END*DIVR2)*DIVR
210 RESULT=RESULT+ADD
211 END=END*DIVR
212 END=END*DIVR
213 DD 216 J=1,L
214 ADD = VALUE(A1,A2,A3,A4,A5,A6,END*DIVR2)*DIVR
215 RESULT=RESULT+ADD
216 END=END*DIVR
217 GO TO 220
218 END=END*DIVA
219 DIVR=DIVA
220 IF (I=1) 222, 222, 221
221 IF I=(N-1) 224, 229, 229
222 RESULT=RESULT+F(I1)*DIVR
223 GO TO 233
224 RESULT=RESULT+F(I1)*DIVR
225 F(I1)=F(2)
226 F(2)=F(3)
227 F(I1)=VALUE(A1,A2,A3,A4,A5,A6,END*(3./Z2.)*DIVA)
228 GO TO 233
229 IF (I=N) 230, 232, 232
230 RESULT=RESULT+F(I1)*DIVR
231 GO TO 233
232 RESULT=RESULT+F(I1)*DIVR
233 Riemann=Riemann*RESULT
234 RETURN, END
C**********SOLUTION TO CASE 4 PROBLEM************

\[ \Theta(X,Y) = \frac{Y \times (X + 1)}{(X + 1)} \]

1 FORMAT(8F10.4)
2 FORMAT(12X,'F10.4')
112 FORMAT(1X,'HEAT FLUX AT ROOT =',F10.2,' BTU/HR-SQFT')
114 FORMAT(20X,'X =',F8.3,' FT',4X,'Y =',F8.2,' FT')
121 FORMAT(11X,'FIN LENGTH =',F8.3,' FT')
122 FORMAT(11X,'T INFINITY =',F8.2,' R')
123 FORMAT(11X,'EMITTANCE =',F8.3)
124 FORMAT(11X,'FIN HALF THICKNESS =',F12.7,' FT')
125 FORMAT(11X,'EXPONENT =',F7.4,'/')
126 FORMAT(11X,'ROOT TEMP. =',F8.2,' R')
127 FORMAT(11X,'CONDUCTIVITY =',F7.2)
401 FORMAT(11X,'CASE 4A PROBLEM//')
402 FORMAT(11X,'CASE 4B PROBLEM//')
403 FORMAT(11X,'CASE 4C PROBLEM//')
404 FORMAT(11X,'MINIMUM TEMP. OF',F8.2,' R OCCURS AT',F8.3,' FT/')
405 FORMAT(11X,'FINITE LENGTH FIN WITH BOTH END TEMPERATURES SPECIFIED//')
406 FORMAT(11X,'TEMP. AT OUTER END =',F8.2,' DEG.R')
413 FORMAT(20X,'FLUX AT OUTER END =',F10.2,' BTU/HR-SQFT')
1001 FORMAT(11X,'N =',I4)
1002 FORMAT(11X,'EPS =',F9.7)
1003 FORMAT(11X,'ACC =',F9.7)
READ(1,2) ITIMES
4 DO 1000 I=1,ITIMES
   READ(1,1) TO,TINF,DEL,E,A,KGO,XL,TL
   READ(1,2) XNO,N,EPS,ACC,ITYPE
   SIGMA=0.1714E-08
   GAMMA=(A+5.1)/(A+1.)
   R=E*SIGMA/(DEL*GAMMA)/(KGO*DEL*(A+5.1))**2.
   G=2.*E*SIGMA*KGO/(DEL*(A+5.1))
   A1 = 1.
   A2 = 1.01
   A3 = 0
   A4 = 0
   A5 = -1.
   An = GAMMA
   R = RIEMAN(A1,A2,A3,A4,A5,A6,N,EPS)
   A1 = A2
   A2 = THETA(A1,TO)/THETA(A1,TL)
   DUM = 1./SQRT(1+THETA(A1,TL)**1.(GAMMA-2.)
   IF(A2.LT.1.01) R = 0
   IF(A2.LT.1.01) A1 = 1.
   IF(ABS(A2-A1).LT.0.001) GO TO 435
   Y = R + RIEMAN(A1,A2,A3,A4,A5,A6,N,EPS)
   XL2 = DUM*Y
   IF(ABS(XL2-2.*XL1).LT.ACC) GO TO 450
   IF(XL2.GT.XL) GO TO 420
   IF(XL2.LT.XL) GO TO 435
435 WRITE(3,402)
450 WRITE(3,405)

56
TABLE XIII (cont)

\[
\begin{align*}
\text{WRITE}(3,121) & \text{XL} \\
\text{WRITE}(3,126) & \text{TG} \\
\text{WRITE}(3,406) & \text{TL} \\
\text{WRITE}(3,122) & \text{TINF} \\
\text{WRITE}(3,123) & \text{TF} \\
\text{WRITE}(3,124) & \text{DEL} \\
\text{WRITE}(3,127) & \text{XKO} \\
\text{WRITE}(3,125) & \text{A} \\
\text{GO TO} & 460 \\
420 \text{ WRITE}(3,401) & \\
\text{WRITE}(3,405) & \\
\text{WRITE}(3,121) & \text{XL} \\
\text{WRITE}(3,126) & \text{TG} \\
\text{WRITE}(3,406) & \text{TL} \\
\text{WRITE}(3,122) & \text{TINF} \\
\text{WRITE}(3,123) & \text{TF} \\
\text{WRITE}(3,124) & \text{DEL} \\
\text{WRITE}(3,127) & \text{XKO} \\
\text{WRITE}(3,125) & \text{A} \\
A1 & = 1. \\
A5 & = 0 \\
A7 & = 1.01 \\
W & = (A2-1)\times2/1\times2/8\times\text{THETA}(A,T,\text{TL})\times(\text{GAMA}-2)-1 \\
421 \ W & = 0.98W \\
A5 & = A5 + W \\
424 \ R & = \text{RIEMAN}(A1,A7,A3,A4,A5,A6,N,\text{EPS}) \\
\text{IF}(\text{A2} \leq \text{LT} \leq 1.01) & A7 = 1. \\
\text{IF}(\text{A2} \leq \text{LT} \leq 1.01) & R = 0 \\
XL & = \text{DUM}(R \times \text{RIEMAN}(A7,A2,A3,A4,A5,A6,N,\text{EPS})) \\
\text{IF}(A5(XL-\text{XL}) \times \text{LT} \leq \text{ACC}) & \text{GO TO} 460 \\
\text{IF}(XL2-\text{XL} \leq 0.005) & \text{GO TO} 421 \\
423 \ A5 & = A5 - W \\
\text{GO TO} & 421 \\
460 \text{ WRITE}(3,114)A3, \text{TO} \\
\text{GO TO} & 460 \\
XNO & = \text{FLOAT}(\text{IXNO}) \\
L & = 1 \\
343 \ T & = \text{IFO} \times \text{L} \times \text{XNO} \\
L & = L + 1 \\
\text{IF}(T \geq (TO-19)) & \text{GO TO} 343 \\
\text{IF}(T \leq (TO+19)) & \text{GO TO} 370 \\
A1 & = \text{THETA}(A,1) / \text{THETA}(A,\text{IL}) \\
A7 & = 1.01 \\
A & = 0 \\
\text{IF}(\text{A1} \geq 1.005) & \text{GO TO} 451 \\
R & = \text{RIEMAN}(A1,A7,A3,A4,A5,A6,N,\text{EPS}) \\
A1 & = A7 \\
451 \ Y & = R \times \text{RIEMAN}(A1,A2,A3,A4,A5,A6,N,\text{EPS}) \\
X & = \text{DUM} \times Y \\
\text{WRITE}(3,114) & X,T \\
\text{GO TO} & 343
\end{align*}
\]

59
370 CONTINUE
WRITE(3,114) XL2,TL
PL2 = (6*THETA(A,TL)**GAMA)*(A5 + 1.)
QD = SQR(TG*(T0**((A+5.1)-TL**((A+5.1)) + XK0**2*PL2)
WRITE(3,112) CQ
QL = XK0*SQR(PL2)
WRITE(3,413) QL
WRITE(3,1001) N
WRITE(3,1002) EPS
WRITE(3,1003) ACC
GO TO 1000
435 WRITE(3,403)
WRITE(3,405)
WRITE(3,121) XL
WRITE(3,126) T0
WRITE(3,406) TL
WRITE(3,122) TINF
WRITE(3,123) E
WRITE(3,124) DEL
WRITE(3,127) XK0
WRITE(3,125) A
TM = 0
W = TL
A7 = 1.01
439 W = .5*W
TM = TM + W
X = L*SQRT(B*THETA(A,TM)**(GAMA-2))
A1 = 1
R1 = 0
R2 = 0
A2 = THETA(A,T0)/THETA(A,TM)
AB = THETA(A,TL)/THETA(A,TM)
IF(A2*LT.1.015) GO TO 452
R1 = Riemann(A1,A7,A3,A4,A5,A6,N,EPS)
A1 = A7
452 Y = X*(R1 + Riemann(A1,A2,A3,A4,A5,A6,N,EPS))
A1 = 1
IF(A8*LT.1.015) GO TO 453
R2 = Riemann(A1,A7,A3,A4,A5,A6,N,EPS)
A1 = A7
453 Z = X*(R2 + Riemann(A1,A8,A3,A4,A5,A6,N,EPS))
Y = Y + Z
IF(AABS(XL-Y),LT,ACC) GO TO 436
IF(YXL-347,436,439
437 TM = TM - W
GO TO 439
436 XM = XL - Z
WRITE(3,404) TM,XM
WRITE(3,114) A3,T0
T0 = IFIX(T0+100)/100.
T0 = 100*T0
XNO = FLOAT(XND)
TABLE XIII (cont)

373 T = TTO-L*XNO
L = L + 1
IF(A.GT.(TTO-10)) GO TO 373
IF(T.LT.(TTO+10)) GO TO 372
AI = THETA(A,1)/THETA(A,1)
RI = 0
IF(A1.GT.1.005) GO TO 457
RI = RIEMAN(A1,A7,A3,A4,A5,A6,N,EPS)
AI = A7
457 Z = X*R1 + RIEMAN(A1,A2,A3,A4,A5,A6,N,EPS))
WRITE(3,116) Z,T
GO TO 373
372 CONTINUE
WRITE(3,114) XN,TM
IF(TM*GT.100.) GO TO 455
IT0 = 0
GO TO 456
455 IT0 = INT((TM-100)/100.)
456 IT0 = 100*IT0
XNO = FLOAT(XNO)
L = 1
454 T = TTO + L*XNO
L = L + 1
IF(T.LT.(TM+10)) GO TO 454
IF(T.GT.(TM-10)) GO TO 371
AI = 1
A2 = THETA(A,1)/THETA(A,1)
RI = 0
IF(A2.GT.1.015) GO TO 458
RI = RIEMAN(A1,A7,A3,A4,A5,A6,N,EPS)
AI = A7
458 Z = X*(R1 + RIEMAN(A1,A2,A3,A4,A5,A6,N,EPS))
Z = Z + XM
WRITE(3,114) Z,T
GO TO 454
371 WRITE(3,114) Y,TL
447 QD = SQRT((G*(i+5.1)*TM **(A5.))/QL
QL = SQRT(G*(i+5.1)*TM **(A5.))
WRITE(3,112) QD
WRITE(3,41)QL
1000 CONTINUE
WRITE(3,1001) N
WRITE(3,1002) EPS
WRITE(3,1003) ACC
STOP
END
FUNCTION RIEMAN(A1,A2,A3,A4,A5,A6,N,EPS)
DIMENSION A1(3),F(A)
VALUE(A1,A2,A3,A4,A5,A6,N,X)=1./(SQRT(X**A6-A3*A6-A4*A6*(X-A3)+A5))
160 DIVA=(A2-A1)/FLOAT(N)
101 UP 103 I=1,3

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107  A(I)=A(I)+(FLOAT(2*I-1)/2.0)*DIVA
108  F(I) = VALUE(A1,A2,A3,A4,A5,A6,A(I))
109  END=A1
110  RIESMA=0.0
119  DD 23J  J=1,N
191  RESULT=0.0
200  ERROR=(((F(3)-2.0*F(2)+F(I))#DIVA)/24.0)*FLOAT(N)
201  R3=ABS(ERROR/EP1)
202  F(ERR=1.0) 218,218,203
203  K=50*F(ERR)+1.0
204  L=K/2
205  K=7*K+1
206  DIVR=DIVA/FLOAT(K)
207  DIVR=DIVR/2.0
208  DD 211  J=1,L
209  ADD=VALUE(A1,A2,A3,A4,A5,A6,END*DIVR2)*DIVR
210  RESULT=RESULT+ADD
211  END=END+DIVR
212  ADD=VALUES(A1,A2,A3,A4,A5,A6,END*DIVR2)*DIVR
213  RESULT=RESULT+ADD
214  END=END+DIVR
217  GO TO 220
218  END=END+DIVA
219  DIVR=DIVA
220  IF(I-1) 222,222,221
221  IF(I-N+1) 224,229,229
222  RESULT=RESULT+F(I)*DIVR
223  GO TO 233
224  RESULT=RESULT+F(I)*DIVR
225  F(I)=F(I)
226  F(I)=F(I)
227  F(3)=VALUE(A1,A2,A3,A4,A5,A6,END+3.0)*DIVA
228  GO TO 233
229  IF(I-N+1) 230,232,232
230  RESULT=RESULT+F(I)*DIVR
231  GO TO 233
232  RESULTS=RESULT+F(I)*DIVR
233  RIESMA=RIESMA+RESULT
234  RETURN
END

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TABLE XIV

CASE V COMPUTER PROGRAM

C*******SOLUTION TO CASE 5 PROBLEM*********

THEAT(X,Y) = (*(X + 1.1)/(X + 1.1)

1 FORMA$TF10,6$

2 FORMAL$TF10,4*$101$

501 FORMAL$TF10,6$ CASE 5 PROBLEM, P/7

112 FORMAL$TF10,4*$HEAT FLUX AT ROOT = $TF10,2,* 6000/HR-SQFT)

116 FORMAL$TF10,4*$MX = $TF10,4* FT, 3X.6FT = $TF10,4* DEC, R)

121 FORMAL$TF10,4*$INFINITE LENGTH FIN)

122 FORMAL$TF10,4*$INFINITY = $TF10,4* FT, 2, 4 R

124 FORMAL$TF10,4*$FIN HALF THICKNESS = $TF10,4* FT, 2, 4 R

125 FORMAL$TF10,4*$EXPONENT = $TF10,4* FT, 4, 4 R

126 FORMAL$TF10,4*$ROOT TEMP = $TF10,4* FT, 2, 4 R

127 FORMAL$TF10,4*$CONDUCTIVITY = $TF10,4* FT, 2, 4 R

REAL(1,2) TIMES

4 DO 100 IU=1, ITIMES

READ(1,1) TO, INF, DEL, E, A, XKO, XL, TL

READ(1,2) IIND, N, EPS, AGC, TYPE

WRITE(1,301)

WRITE(1,121)

WRITE(1,126) TO

WRITE(1,122) INF

WRITE(1,123) E

WRITE(1,124) DEL

WRITE(1,127) XKO

WRITE(1,128) A

STGMA=0.1745E-08

GAMA=1.4517E+1

DE1=SIGMA*FL10,4)

A=DE1*SIGMA*(A+1.1)**GAMA/((XKO*DEL*(A+5.1))**2*

C=2.*E*SIGMA*XKO/DEL*(A+5.1)

X = 1./SQRT((A**2+(THETA*A)*INF)**(GAMA-2.1))

A2 = THETA(A, TO)/THETA(A, INF)

A3 = 1.

A4 = 1.

A5 = 0.

A6 = GANA

WRITE(1,144) A5, TO

TO = FIX(TO+100)/100.

TTO = 100*TO

XNO = FLOAT(XKO)

L = 1

341 Y = TTO-L*XNO

L = L + 1

IF(LT(TO-101)) GO TO 343

IF(LT((TO+10))) GC TO 370

A1 = THETA(A, TO)/THETA(A, INF)

Y = RIKMA(A1, A2, A3, A4, A5, A6, EPS)

XL = X*Y

WRITE(1,144) XLL, T

GO TO 343

370 CONTINUE

END

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FUNCTION Riemann (A1, A2, A3, A4, A5, A6, N, EPS)
VALUE(A1, A2, A3, A4, A5, A6, K) = 1 / SQRT((A6 - A3) ** 2 + (A4 - A1) ** 2)
DIVA = (A2 - A1) / FLOAT(N)
DO 103 I = 1, N
103 A(I) = A1 + FLOAT((I - 1) / N) * DIVA
104 END = A(1)
105 RTN = 0
199 DO 233 I = 1, N
199 RESULT = 0
200 ERROR = 0
201 ERR = ABS(ERROR / EPS)
202 IF (ERR > 1.0) GOTO 218, 218, 203
203 K = SQRT(ERR) + 1.0
204 L = K / 2
205 K = 2 * L + 1
206 DIVR = DIVA / FLOAT(K)
207 DIVR2 = DIVR / 2.0
208 DO 211 J = 1, L
209 ADD = VALUE(A1, A2, A3, A4, A5, A6, END + DIVR2) * DIVR
210 RESULT = RESULT + ADD
211 END = END + DIVR
212 END = END + DIVR
213 DO 216 J = 1, L
214 ADD = VALUE(A1, A2, A3, A4, A5, A6, END + DIVR2) * DIVR
215 RESULT = RESULT + ADD
216 END = END + DIVR
217 GO TO 220
218 END = END + DIVA
219 DIVR = DIVA
220 IF (I < 1) GOTO 222, 222, 221
221 IF (I > N) GOTO 224, 229, 229
222 RESULT = RESULT + F(I) * DIVR
223 GO TO 233
224 RESULT = RESULT + F(2) * DIVR
225 F(1) = F(2)
226 F(2) = F(3)
227 F(3) = VALUE(A1, A2, A3, A4, A5, A6, END + 3.0 / 2.0 * DIVA)
228 GO TO 233
229 IF (I < N) GOTO 230, 232, 232
230 IF (I < N) GOTO 230, 232, 232
231 IF (I < N) GOTO 230, 232, 232
232 IF (I < N) GOTO 230, 232, 232

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230  RESULT = RESULT + F(2) * DIVR
231  GO TO 231
232  RESULT = RESULT + F(3) * DIVR
233  RIEMAN = RIEMAN + RESULT
234  RETURN
END
CASE VI COMPUTER PROGRAM

SOLUTION TO CASE 6 PROBLEM

\[ \theta_{\text{theta}}(x,y) = \left( y^{**}(x + 1.1)/(x + 1.1) \right) \]

1. FORMAT(8,10,4) 10
2. FORMAT(21,10,4) 10
112 FORMAT(//,20X,'HEAT FLUX AT ROOT =',F10.2,' BTU/HR-FT2.Special(*')
114 FORMAT(20X,'HX =',F8.3,' FT-3X4HT =',F8.2,' DEG.R.')
122 FORMAT(11X,'TIME =',F8.2,' R+1')
123 FORMAT(11X,'EMITTANCE =',F8.3)
124 FORMAT(11X,'FIN LENGTH =',F8.3,' FT+1')
125 FORMAT(11X,'EXPONENT =',F7.4'/')
126 FORMAT(11X,'ROOT TEMP =',F8.2,' R+1')
127 FORMAT(11X,'CONDUCTIVITY =',F7.2)"
TABLE XV (cont.)

```fortran
650 WRITE(3,207) TL
   WRITE(3,114) A5,TO
   ITO = FIX(TO*(TO+100)/100.)
   TTO = 100*TO
   XNO = FLOAT(TXNO)
   L = I
343 T = ITO-L*XNO
   L = L + 1
   IF(T<GT.(TO-10)) GO TO 343
   IF(TL>T(1L+1C)) GO TO 370
   A1 = (T/TINF)**(A+1)
   XL2 = X*Riemann(A1,A2,A3,A4,A5,A6,N,EPS)
   WRITE(3,114) XL2,I
   GO TO 343
370 CONTINUE
   WRITE(3,114) XL1,IL
   DO = SORTG(TO**(A+1.5)-(A+5.)/(A+1.5)**TINF)**4*(TO**(A+1.5)**(A+1.5)**TINF)**4*(T**A+1.1)
   WRITE(3,122) DO
   WRITE(3,1001) N
   WRITE(3,1002) EPS
   WRITE(3,1003) ACC
1001 FORMAT(11X,N="",I4)
1002 FORMAT(11X,EPS="",F9.7)
1003 FORMAT(11X,ACC="",F9.7)
1000 CONTINUE
   STOP
END
FUNCTION Riemann(A1,A2,A3,A4,A5,A6,N,EPS)
   DIVA = 100.0
  (value = 1.0)
   IF(X>1.0) THEN
      Riemann = 1.0
   ELSE
      Riemann = X
   END IF
END
DIVA = A2-A1/FLOAT(N)
100 DIVA = A2-A1/FLOAT(N)
103 I=1
104 A=1
105 A=1
106 A=1
108 A=1
110 A=1
111 A=1
119 DO 233 I=1,N
199 RESULT = 0.0
200 ERR= ( (F(3)-2.0*F(2)+F(1))*DIVA/2.0 FLOATE(N)
201 ERR= ( (F(3)-2.0*F(2)+F(1))*DIVA/2.0 FLOATE(N)
202 IF(ERR>1.0) THEN
203 K = SQRT(ERR)*1.0
204 L = K/2
205 K = 2*K+1
206 DIVA = DIVA/FLOAT(N)
207 DIVA = DIVA/FLOAT(N)
208 GO TO 211
209 ADD = VALUE(A1,A2,A3,A4,A5,A6,N,END*DIVA2)*DIVA
210 RESULT = RESULT + ADD
211 END = END + DIVA
212 END = END + DIVA

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```
213 DD 216 J=1,L
214 ADD = VALUE(A1,A2,A3,A4,A5,A6,END+DIVR2)*DIVR
215 RESULT=RESULT+ADD
216 END=END+DIVR
217 GO TO 220
218 END=END+DIVA
219 DIVR=DIVA
220 IF(I-1) 222,222,221
221 IF(I-(N-1)) 224,229,229
222 RESULT=RESULT+F(1)*DIVR
223 GO TO 233
224 RESULT=RESULT+F(2)*DIVR
225 F(1)=F(2)
226 F(2)=F(3)
227 F(3) = VALUE(A1,A2,A3,A4,A5,A6,END*(3./2.))*DIVA
228 GO TO 233
229 IF(I-N) 230,232,232
230 RESULT=RESULT+F(2)*DIVR
231 GO TO 233
232 RESULT=RESULT+F(3)*DIVR
233 RIEMAN=RIEMAN+RESULT
234 RETURN
END
TABLE XVI

CASE VII COMPUTER PROGRAM

C**********SOLUTION TO CASE 7 PROBLEM**********

THETA(I,Y) = (Y**2*(X + 1.1))/(X + 1.)
1 FORMAT(8F10.4)
2 FORMAT(10X,F10.4,I10)
112 FORMAT(2X,20X,HEAT FLUX AT ROOT = *F10.2,* BTU/HR-SQFT)
114 FORMAT(20X,F10.4) FT.*3X,4HT. = *F8.2,* DEG.R)
121 FORMAT(2X,F10.4,F30.10) F11)
122 FORMAT(1X,*E FN LENGTH = *F8.3,* FT*)
123 FORMAT(1X,*E FERMATICE = *F8.3)
124 FORMAT(1X,*E FIN HALF THICKNESS = *F12.7,* FT*)
125 FORMAT(1X,*E EXPOEND = *F7.6)
126 FORMAT(1X,*E ROOT TEMP. = *F8.2,* R*)
127 FORMAT(1X,*E CONDUCTIVITY = *F7.2)
404 FORMAT(1X,*E MINIMUM TEMP. OF *F8.2,* R OCCURS AT *F12.7,* FT*)
405 FORMAT(1X,*E FINE UT LENGTH FIN WITH BOTH END TEMPERATURES SPECIFIED

1)^ 406 FORMAT(1X,*E TEMP. AT OUTER END = *F8.2,* DEG. R*)
411 FORMAT(20X,*E HEAT FLUX AT OUTER END = *F10.2,* BTU/HR-SQFT)
701 FORMAT(1X,*E CASE 7 PROBLEM***/)
704 FORMAT(1X,*E CASE 7 PROBLEM***/)
705 FORMAT(20X,*E TEMP AT END OF FIN EQUALS T INF*)
708 FORMAT(1X,*E CASE 7 PROBLEM***/)
777 FORMAT(20X,*E HEAT FLUX AT INFLATION POINT = *F10.2,* BTU/HR-SQFT)
778 FORMAT(20X,*E INFLATION POINT OCCURS AT *F12.7,* FT*)
1001 FORMAT(1X,*E N = *I4)
1002 FORMAT(1X,*E EPS = *F9.7)
1003 FORMAT(1X,*E ACC = *F9.7)
READ(1,2) TIMES
4 BU LOC1 IDA=1,ITIMES
READ(1,2) TO,TINF,DEL,E,A,XKO,XTL
READ(1,2) INNO,N,EPSS,ACC,ITYPE
SIGMA=0.1714E-08
GAMMA=1.514E+08
B=(EPSIGMA*(A+1.))/GAMMA/(XKO*DEL*(A+5.))**2.
G=2.*EPSIGMA*XKO/(DEL*(A+5.))
IF(ABS(TIL-TINF).LT.1.1JGOTO 740
IF(TIL-TINF).LT.7GOTO 702
IF(TIL-TINF).GT.7GOTO 760
702 WRITE(3,701)
B2=THETA(A,TIL)/THETA(A,TINF)
X=1.-GAMMA*GAMMA*B2**GAMMA
IF(LT,0.01)=ABS(Z)
IF(ABS,.GT,-0.0) LMA=0.
703 CONTINUE
IF(LMN.NE.1.)WRITE(3,705)
WRITE(3,405)
WRITE(3,121)T
WRITE(3,126)T
WRITE(3,406) T
WRITE(3,122)T
WRITE(3,123)T

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WRITE(3,124)DEL
WRITE(3,127)XKO
WRITE(3,129)A
K=1000.
A6=GAMA
B=THETA(A,T0)/THETA(A,TINF)
E3=1.
D=1./SQR(R*THETA(A,TINF)**(GAMA-2.))
710 R=R-W
XL=0.*RIEMAN(B2,B3,B2,B3,R,A6,N,EPS)+RIEMAN(B3,B1,B2,B3,R,A6,N,EPS)
1S1)
IF(A8XIXL1-IXL1,ACC)GOTO 720
IF(IXL1-IXL115,720,710)
715 R=R-W
W=.5*W
GOTO 710
720 ALFA = 0
WRITE(3,114)ALFA,TD
ITD=1+IX(XD+100)/100.*1
ITD=100*ITD
XNO = FLOAT(XNO)
L = 1
343 T = ITD - L*XNO
L = L + 1
IF(LLT.LT.(ITD-101) GO TO 343
IF(LLT.(TINF+10)) GO TO 370
B4 = THETA(A,T)/THETA(A,TINF)
XL2=0.*RIEMAN(B4,B1,B2,B3,R,A6,N,EPS)
WRITE(3,114)XL2,T
GO TO 343
370 % = 1
XL2=0.*RIEMAN(B4,B1,B2,B3,R,A6,N,EPS)
XI=XL2
WRITE(3,114)XL2,TINF
IF(LMN.EQ.1) GO TO 373
371 TTD = ITD+(TINF+100)/102.*1
ITD=100*ITD
XNO = FLOAT(XNO)
L = 1
345 T = ITD - L*XNO
L = L + 1
IF(LLT.(TINF-10)) GO TO 345
IF(LLT.(TL+10)) GO TO 372
B4 = THETA(A,T)/THETA(A,TINF)
XL2=0.*RIEMAN(B4,B3,B2,B3,R,A6,N,EPS)
WRITE(3,114)XL2,T
GO TO 345
372 B4 = THETA(A,T)/THETA(A,TINF)
XL2=0.*RIEMAN(B4,B3,B2,B3,R,A6,N,EPS)
WRITE(3,114)XL2,TL
CONTINUE
373 PL2=R***(THETA(A,TINF)**GAMA

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TABLE XV. (cont)

741 WRITE(3,704)
R=0.
B2=1.
LNN=1
GOTO 703

760 WRITE(3,708)
WRITE(3,405)
WRITE(3,121)PL
WRITE(3,126)T
WRITE(3,406) TL
WRITE(3,122)TINF
WRITE(3,123)E
WRITE(3,124)DEL
WRITE(3,127)XKO
WRITE(3,125)T
A2=THETA(A,T0)/THETA(A,TINF)
A1=THETA(A,T0)/THETA(A,TINF)
A3=A1
A4=1.
A5=0.
A6=GAMA

D=1./SQR(B*THETA(A,TINF)**(GAMA-2.))
IF(ALG(A2-A1),LT.,.001) GO TO 781
Z=DERIVMAN(A2,A3,A4,A5,A6,A8,EPS)
IF(XL,GT.,XL-ACC)GOTO 761
IF(Z0,L,LT.,XL-ACC)GOTO 781
WRITE(3,14) A5,TO
ITO = IFIX(TO/1000)/100.
TTO = 100*ITO
YN0 = FLOAT(XNO)
L = 1

346 I = TTO - L*YN0
L = L + 1

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TABLE XVI (cont.)

IF (T > (T+10-1)) GO TO 366
IF (L < (L + 10)) GO TO 374
A1 = THETA(A,T)/THETA(A,TINF)
X12 = D*Riemann(A1,A2,A3,A4,A5,A6,N,EP) WRITE(3,114)X12,T
GO TO 346

374 A1 = THETA(A,T)/THETA(A,TINF)
X12 = D*Riemann(A1,A2,A3,A4,A5,A6,N,EP)
WRITE(3,114)X12,T
GO TO 346

1 = TL**2*(A+L)/(A+5.)*(A+5.)/(A+5.)*TINF**4*(TO**2*(A+L))
QL = 0.
WRITE(3,117) QL
WRITE(3,1001) N
WRITE(3,1002) EPS
WRITE(3,1003) ACC
GOTO 1000

761 W = (TO-TL)/XL*2/(B*THETA(A,TINF)**GAMA)

762 A5 = A5+W
X12 = D*Riemann(A1,A2,A3,A4,A5,A6,N,EP)
IF (ABS(X12-XL),LT,ACC) GOTO 771
IF (X12-XL),763,771,762

763 A5 = A5-W
W = .5*W
GOTO 762

771 ALFA = 0
WRITE(3,114) ALFA, TO
IT0 = FIX((TO+100)/100.)
TTO = 100*IT0
XNO = FLOAT((IT0))
L = 1

350 T = TTO - L*XNO
L = L + 1
IF (T > (T+10-1)) GO TO 350
IF (L < (L + 10)) GO TO 380
A1 = THETA(A,T)/THETA(A,TINF)
X11 = D*Riemann(A1,A2,A3,A4,A5,A6,N,EP)
WRITE(3,114) X11,T
GO TO 350

380 A1 = THETA(A,T)/THETA(A,TINF)
X11 = D*Riemann(A1,A2,A3,A4,A5,A6,N,EP)
WRITE(3,114) X11,T
PL2 = A5*B*THETA(A,TINF)**GAMA
QL = SQRT((T**2*(A+5.)-TL**2*(A+5.)-(A+5.)/(A+5.)*TINF**4*(TO**2*(A+L))
1 = TL**2*(A+L))**2)*XKO**2*PL2)
WRITE(3,112) GD
WRITE(3,113) QL
WRITE(3,1001) N
WRITE(3,1002) EPS
WRITE(3,1003) ACC

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TABLE XVI (cont)

GOTO 1000

781 TM=TINF
    W = TL-TINF

786 W = W + 5*W
    TM = TM + W

787 IF(A>5*(TM-TINF),LT.-.01) TM = TINF
A3=THETA(A1,IM)/THETA(A1,TINF)
XL2=0.0*RIEMAN(A1,A2,A3,A4,A5,A6,N,EPS)+2.0*RIEMAN(A3,A1,A3,A4,A5,A6
1,N,EPS)
IF(A>5*(XL2-XL1).LT.100) GOTO 790
IF(XL2-XL1).LT.105,790,786

788 TM=TM-W
GOTO786

790 XM=0.0*RIEMAN(A3,A2,A3,A4,A5,A6,N,EPS)
WRITE(3,404)TM,XM
ALFA = 0
WRITE(3,114) ALFA,TO
TO = IFIX(TD+100)/100.
L = 10*TO
XNO = FLOAT(XNO)

351 T = TO - L*XNO
L = L + 1
IF(T,GT.(TO-10)) GO TO 351
IF(T,LT.(TM + 10)) GO TO 381
ALFA = THETA(A1,1)/THETA(A1,TINF)
XL1=0.0*RIEMAN(A7,A2,A3,A4,A5,A6,N,EPS)
WRITE(3,114) XL1,T
GO TO 351

381 ALFA = THETA(A1,1)/THETA(A1,TINF)
XL1=0.0*RIEMAN(A7,A2,A3,A4,A5,A6,N,EPS)
WRITE(3,114) XL1,T
TO = IFIX(TM-100)/100.
L = 100*TO
XNO = FLOAT(XNO)

352 T = TO + L*XNO
L = L + 1
IF(T,LT.(TM+10)) GO TO 352
IF(A,LT.(T-L - 10)) GO TO 382
ALFA = THETA(A1,1)/THETA(A1,TINF)
XL1=0.0*RIEMAN(A7,A1,A3,A4,A5,A6,N,EPS)
WRITE(3,114) XL1,T
GO TO 352

382 ALFA = THETA(A1,1)/THETA(A1,TINF)
XL1=0.0*RIEMAN(A7,A1,A3,A4,A5,A6,N,EPS)
WRITE(3,114) XL1,T

796 QT=SQRT(GM(TO**2**(A+5.)*-TM**2**(A+5.))/(A+1.)*TINF**4*(TO**2**(A+1.
+TM**2**(A+1.))))
QL=SQRT(GM(TL**2**(A+5.)*-TM**2**(A+5.))/(A+1.)*TINF**4*(TL**2**(A+1.
+TM**2**(A+1.)))-1.)
WRITE(3,112) QD

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TABLE XVI (cont)

WRITE(3,413)OL
WRITE(3,1001)N
WRITE(3,1002)EPS
WRITE(3,1003)ACC

1C00 CONTINUE
STOP
END

FUNCTION RIEMAN (A1,A2,A3,A4,A5,A6,N,EPS)
DIMENSION A(3),F(3)
VALUE(A1,A2,A3,A4,A5,A6,X)=1./(SQRT(X**A6-A3**A6-A4**A6+(X-A3)**A5))
1D0 DIVA=(A2-A1)/FLOAT(N)
101 DO 103 I=1,3
102 A(I)=A1+(FLOAT(2*I-1)/2.0)*DIVA
103 F(I)=VALUE(A1,A2,A3,A4,A5,A6,A(I))
104 CND=A1
105 RIEMAN=0.0
199 GO TO 233
233 I=1,N

1941 RFSTR=1.0
200 I=NR= 1 IF(F(I)-2.0*F(2)+F(I))>DIVA)/24.0)*FLOAT(N)
201 ERR=ABS(ERR)/EPS)
202 IFERR=-1.0) 218,218,203
204 K=SQRT(ERR)+1.0
206 L=K/2
205 K=2*L+1
206 DIVR=DIVA/FLOAT(K)
207 DIVR2=DIVR/2.0
208 DO 211 J=1,L
209 ADD = VALUE(A1,A2,A3,A4,A5,A6,N,END+DIVR2)*DIVR
210 RESULT=RESULT+ADD
211 END=END+DIVR
212 END=END+DIVR
213 GO TO 216
216 END=END+DIVR
217 GO TO 220
218 END=END+DIVA
219 DIVR=DIVA
220 IF(I=1) 222,222,222
221 IF(I=(N-1) 224,229,229
222 RESULT=RESULT+F(I)*DIVR
223 GO TO 233
224 RESULT=RESULT+F(1)*DIVR
225 F(I)=F(I)
226 F(I+IF(I=1) 227,227,227)
227 F(I)= VALUE(A1,A2,A3,A4,A5,A6,N,END+13,2.)*DIVA)
228 GO TO 233
229 IF(I=N) 230,232,232
230 RESULT=RESULT+F(2)*DIVR
231 GO TO 233
232 RESULT=RESULT+F(3)*DIVR
233 RIEMAN=RESULT

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234 RETURN
END
**SOLUTION TO CASE 8 PROBLEM**

\[ \text{Theta}(x, y) = \left[ y^2(1 + x) + 1 \right] / (x + 1.1) \]

1 FORMAT(B6E4.4)
2 FORMAT(2X,2E10.4,1X,10I1)
112 FORMAT(20X,'HEAT FLUX AT ROOT = ',F10.2,1X,'TU/HR-SQFT')
114 FORMAT(20X,'4X = ',F8.3,4X,'4H FT = ',F8.2,1X,'DEG.R')
122 FORMAT(1X,'INFINITY = ',F8.2,1X,'R')
123 FORMAT(1X,'EXPERIENCE = ',F6.3)
124 FORMAT(1X,'FIN HALF THICKNESS = ',F12.7,1X,'FT')
125 FORMAT(1X,'EXPONENT = ',F7.6,1X,'/')
126 FORMAT(1X,'ROOT TEMP. = ',F8.2,1X,'K')
127 FORMAT(1X,'CONDUCTIVITY = ',F12.2)
206 FORMAT(1X,'FIN LENGTH = ',F8.3,4X,'FT')
207 FORMAT(1X,'TEMPERATURE AT OUTER END = ',F8.2,1X,'DEG. K')
208 FORMAT(1X,'GAMMA = ',F7.6,1X,'/')
413 FORMAT(20X,'FLUX AT OUTER END = ',F10.3,1X,'TU/HR-SQFT')
801 FORMAT(1X,'CASE 8 PROBLEM')
1001 FORMAT(1X,'IN',1X,'N = ',1X,1E1)
1002 FORMAT(1X,'EPS = ',1X,1E7)
1003 FORMAT(1X,'ACC = ',1X,1E7)

READ(1,2) TIMES
4 DO 100 IDA=1,TIMES
READ(1,1) TU,TINF,DEL,F,A,XKO,HL,IL
READ(1,2) INKO,4,EPS,ACC,ITYPE
WRITE(3,201)
WRITE(3,221)
WRITE(3,231)XL
WRITE(3,12610)
WRITE(3,12710)
WRITE(3,12310)
WRITE(3,12410)
WRITE(3,12710)
WRITE(3,12510)
SOMA=(A+5,1)/(A+1,1)
B=EPS*(SOMA**((A+1,1))**GAMMA)/IKKO*DELS+(A+5,1)*2,
C=2.*EPS*(SOMA**XKO)/(DELS+(A+5,1))
Z = 2,
A4 = 1.
A2 = THEATA(A,T10)/THEATA(A,TINF)
A6 = GAMMA
0 = 5*(10-1*TINF)
TL = TINF
X = 1./SQRT(8*THEATA(A,TINF)**(GAMMA-2))
Y = EPS*(SOMA**DELS+(A+5,1))**TINF**((3.-A)/12*IKKO)
821 TL = TL + W
A1 = THEATA(A,TL)/THEATA(A,TINF)
A3 = A1
A5 = XX*(A1**4./(A1**11-1.))**2
Y = X*RIEMAN(A1,A2,A3,A4,A5,A6,EPS)
IFAMS(XL-Y,TL,ACC) GO TO 820
IFLY=XL 810,820,930

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TABLE XVII (cont)

810 TL = TL - W
  W = .5*W
  GO TO 821
830 W = .5*W
  GO TO 921
820 WRITE(3,207) TL
  WRITE(3,114) Z,TO
  TO = IFIX((TO+100.)/100.)*
  TO = 100*TO
  XNO = FLOAT(XNO)
  L = 1
823 T = TO - L*XNO
  L = L + 1
  IF(T.GT.(TO-100)) GO TO 823
  IF(T.LT.(TO+100)) GO TO 870
  A1 = THEATA(A1,T)/THEETA(A1,TINF)
  XLI = X*THEETA(A1,A2,A3,A4,A5,A6,N,EX)
  WRITE(3,114) XLI,T
  GO TO 823
870 WRITE(3,114) Y,TL
  QD = SORT(GE(T0*(A5.5+*A5.5)-A5.*TINF^A5.5*(T0(A5.5-A5.5)-T0(A5.5-A5.5)))*.5)*2
  QL = E*SIGMA(TL*(1+*TINF)^4.44)
  WRITE(3,112) QD
  WRITE(3,413) QL
  WRITE(3,1001) N
  WRITE(3,1002) EPS
  WRITE(3,1003) ACC
100 CONTINUE
STOP
END
FUNCTION RIEMAN(A1,A2,A3,A4,A5,A6,N,EX)
DIMENSION A1(3,F(1))
VALUE(A1,A2,A3,A4,A5,A6,X)=1./SORT(KK66-A3*A6-A4*A6-(X-A3)*A5))
100 DIVA=A2-A1)/FLOAT(4)
101 ON 103 F=1.3
102 A1=A1+(FLOAT(2*[-1/2,0])*DIVA)
103 F(1) = VALUE/A1,A2,A3,A4,A5,A6,A(1))
104 END=A1
105 RIEMAN=0.0
199 DO 233 F=1,N
200 ERROR = ((F.F3.[-2*F2+F1]1)1)*DIVA/24.0)*FLGAT(N)
201 ERRABS(ERG,F/EP)
207 IF(ERG<1.0) 218,219,203
203 K=SQRT(FERG)*1.0
204 L=X/K
205 K=2*X+1
206 DIVA=DIVA/FLCAT(1)
207 DIVA=DIVA/2.0
209 ON 211 J=1,L
209 AND = VALUE/A1,A2,A3,A4,A5,A6,END+DIVA2)*DIVA

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210  RESULT=RESULT+ADD
211  END=END+DIVR
212  END=END+DIVR
213  DD 216 31,L
214  ADD = VALUE(A1,A2,A3,A4,A5,A6,END+DIVR21)*DIVR
215  RESULT=RESULT+ADD
216  END=END+DIVR
217  GO TO 220
218  END=END+DIVA
219  DIVR=DIVA
220  IF(1-1) 222,222,221
221  IF(1-(N-1)) 224,229,229
222  KRESULT=RESULT+F111*DIVR
223  GO TO 233
224  RESULT=RESULT+F(1)*DIVR
225  F(1)=F(2)
226  F(2)=F(3)
227  F(3) = VALUE(A1,A2,A3,A4,A5,A6,END+(3/2)*DIVA)
228  GO TO 233
229  IF(1-N) 230,232,232
230  RESULT=RESULT+F(2)*DIVR
231  GO TO 233
232  RESULT=RESULT+F(1)*DIVR
233  RIEMAN=RIE MAN+RESULT
234  RETURN
END
TABLE XVIII
CASE I SAMPLE OUTPUT

CASE I PROBLEM

INFINITE LENGTH FIN
RCO TEMP = 530.00 R
T INFINITY = 0.00 R
EMITTANCE = 0.950
FIN HALF THICKNESS = 0.0208333 FT
CONDUCTIVITY = 29.37
EXponent = 2.1980

<table>
<thead>
<tr>
<th>X</th>
<th>T</th>
<th>Heat Flux at RCCT = 11309.14 BTU/HR-SCFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.500 FT.</td>
<td>530.00 DEG.R</td>
<td></td>
</tr>
<tr>
<td>0.289 FT.</td>
<td>500.00 DEG.R</td>
<td></td>
</tr>
<tr>
<td>0.476 FT.</td>
<td>453.03 DEG.R</td>
<td></td>
</tr>
<tr>
<td>1.344 FT.</td>
<td>400.00 DEG.R</td>
<td></td>
</tr>
<tr>
<td>2.482 FT.</td>
<td>350.00 DEG.R</td>
<td></td>
</tr>
<tr>
<td>4.140 FT.</td>
<td>300.00 DEG.R</td>
<td></td>
</tr>
<tr>
<td>6.346 FT.</td>
<td>250.00 DEG.R</td>
<td></td>
</tr>
<tr>
<td>9.924 FT.</td>
<td>200.00 DEG.R</td>
<td></td>
</tr>
<tr>
<td>16.538 FT.</td>
<td>150.00 DEG.R</td>
<td></td>
</tr>
<tr>
<td>31.789 FT.</td>
<td>100.00 DEG.R</td>
<td></td>
</tr>
<tr>
<td>89.532 FT.</td>
<td>50.00 DEG.R</td>
<td></td>
</tr>
</tbody>
</table>

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### TABLE XX
CASE III SAMPLE OUTPUT

**CASE 3 PROBLEM**

| **FINITE LENGTH FIN WITH RADIATING EN\\** | **FIN LENGTH = 3,000 FT.** |
| **RECT TEMP. = 1000.00 R** |
| **T INFINITY = 0.00 R** |
| **EMITANCE = 0.950** |
| **FIN HALF THICKNESS = 0.0208333 FT** |
| **CONDUCTIVITY = 100.00** |
| **EXONENT = 0.0000** |

**TEMPERATURE AT OUTER EN\\** = 536.76 DEG. R

<table>
<thead>
<tr>
<th>X</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000 FT.</td>
<td>1000.00 DEG.R</td>
</tr>
<tr>
<td>0.098 FT.</td>
<td>950.00 DEG.R</td>
</tr>
<tr>
<td>0.210 FT.</td>
<td>900.00 DEG.R</td>
</tr>
<tr>
<td>0.341 FT.</td>
<td>850.00 DEG.R</td>
</tr>
<tr>
<td>0.494 FT.</td>
<td>800.00 DEG.R</td>
</tr>
<tr>
<td>0.678 FT.</td>
<td>750.00 DEG.R</td>
</tr>
<tr>
<td>0.904 FT.</td>
<td>700.00 DEG.R</td>
</tr>
<tr>
<td>1.191 FT.</td>
<td>650.00 DEG.R</td>
</tr>
<tr>
<td>1.583 FT.</td>
<td>600.00 DEG.R</td>
</tr>
<tr>
<td>2.237 FT.</td>
<td>550.00 DEG.R</td>
</tr>
<tr>
<td>2.949 FT.</td>
<td>500.76 DEG.R</td>
</tr>
</tbody>
</table>

**HEAT FLUX AT RECT** = 54723.61 BTU/HR-SCFT

**FLUX AT OUTER EN\\** = 129.22 BTU/HR-SCFT

N = 25
EPS = 0.930000
ACC = 0.001000

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CASE 4A PROBLEM

FINITE LENGTH FIN WITH BOTH END TEMPERATURES SPECIFIED
FIN LENGTH = 3.00 FT
ROOT TEMP. = 1000.00 R
TEMP. AT OUTER END = 400.00 DEG.R
T INFINITY = 750.00 R
EMITTANCE = 0.60
FIN HALF THICKNESS = 0.0213333 FT
CONDUCTIVITY = 17.50
EXPOSURE = 0.7500

\[ \begin{align*}
  x &= 0.764 \text{ FT}, \quad T = 1000.00 \text{ R} \\
  x &= 0.137 \text{ FT}, \quad T = 950.00 \text{ R} \\
  x &= 0.290 \text{ FT}, \quad T = 900.00 \text{ R} \\
  x &= 0.461 \text{ FT}, \quad T = 850.00 \text{ R} \\
  x &= 0.653 \text{ FT}, \quad T = 800.00 \text{ R} \\
  x &= 0.887 \text{ FT}, \quad T = 750.00 \text{ R} \\
  x &= 1.106 \text{ FT}, \quad T = 700.00 \text{ R} \\
  x &= 1.368 \text{ FT}, \quad T = 650.00 \text{ R} \\
  x &= 1.656 \text{ FT}, \quad T = 600.00 \text{ R} \\
  x &= 1.966 \text{ FT}, \quad T = 550.00 \text{ R} \\
  x &= 2.296 \text{ FT}, \quad T = 500.00 \text{ R} \\
  x &= 2.642 \text{ FT}, \quad T = 450.00 \text{ R} \\
  x &= 2.999 \text{ FT}, \quad T = 400.00 \text{ R}
\end{align*} \]

HEAT FLUX AT ROOT = 38644.31 BTU/HR-FT²
FLUX AT OUTER END = 138724.48 BTU/HR-FT²

N = 25
EPS=0.300000
ACC=0.001000
TABLE XXII
CASE IVb SAMPLE OUTPUT

CASE 4b PRCELEM

FINITE LENGTH FIN WITH BOTH END TEMPERATURES SPECIFIED
FIN LENGTH = 3.000 FT
RECT TEMP. = 1000.00 R
TEMP. AT OUTER END = 532.84 DEG.R
T INFINITY = 0.00 R
EPI TANGENCE = 0.950
FIN HALF THICKNESS = 0.0208333 FT
CONDUCTIVITY = 100.00
EXPONENT = 0.0000

\[
\begin{align*}
X &= 0.000 \text{ FT, } T = 1000.00 \text{ R} \\
X &= 0.208 \text{ FT, } T = 950.00 \text{ R} \\
X &= 0.341 \text{ FT, } T = 856.00 \text{ R} \\
X &= 0.495 \text{ FT, } T = 800.00 \text{ R} \\
X &= 0.679 \text{ FT, } T = 750.00 \text{ R} \\
X &= 0.705 \text{ FT, } T = 700.00 \text{ R} \\
x &= 1.194 \text{ FT, } T = 650.00 \text{ R} \\
x &= 1.589 \text{ FT, } T = 600.00 \text{ R} \\
x &= 2.261 \text{ FT, } T = 550.00 \text{ R} \\
x &= 3.000 \text{ FT, } T = 532.84 \text{ R}
\end{align*}
\]

HEAT FLUX AT RCCT = 54699.71 BTU/HR-SQFT
FLUX AT OUTER END = 0.00 STU/HR-SQFT

N = 25
EPS=0.000000010
ACC=0.0010000

N = 25
EPS=0.00000010
ACC=0.00100000

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CASE AC PROBLEM

FINITE LENGTH FIN WITH BOTH END TEMPERATURES SPECIFIED
FIN LENGTH = 2,000 FT
RECT TEMP. = 800.00 R
TEMP. AT OUTER END = 800.00 DEG.R
T INFINITY = 0.00 R
EMITTANCE = 1.000
FIN HALF THICKNESS = 0.0208333 FT
CONDUCTIVITY = 50.00
EXPONENT = 0.0000

MINIMUM TEMP. OF 639.84 R OCCURS AT 1,000 FT

<table>
<thead>
<tr>
<th>X</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>800.00</td>
</tr>
<tr>
<td>0.149</td>
<td>750.00</td>
</tr>
<tr>
<td>0.354</td>
<td>700.00</td>
</tr>
<tr>
<td>0.725</td>
<td>650.00</td>
</tr>
<tr>
<td>1.000</td>
<td>639.84</td>
</tr>
<tr>
<td>1.274</td>
<td>650.00</td>
</tr>
<tr>
<td>1.645</td>
<td>700.00</td>
</tr>
<tr>
<td>1.950</td>
<td>750.00</td>
</tr>
<tr>
<td>2.000</td>
<td>800.00</td>
</tr>
</tbody>
</table>

HEAT FLUX AT RECT = 19045.10 BTU/HR-SQFT
FLUX AT OUTER END = -19045.10 BTU/HR-SQFT
TABLE XXIV
CASE V SAMPLE OUTPUT

CASE 5 PROBLEM

INFINITE LENGTH FIN
ROOT TEMP. = 1000.00 R
T (INFINITY) = 169.60 R
EMITTANCE = 0.900
FIN HALF THICKNESS = 0.0208333 FT
CONDUCTIVITY = 100.00
EXPERIMENT = 5.000

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000</td>
<td>0.147</td>
<td>95.405</td>
</tr>
<tr>
<td>0.315</td>
<td>0.416</td>
<td>90.700</td>
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<tr>
<td>0.518</td>
<td>0.732</td>
<td>85.000</td>
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<tr>
<td>1.732</td>
<td>1.494</td>
<td>75.000</td>
</tr>
<tr>
<td>1.524</td>
<td>1.675</td>
<td>65.000</td>
</tr>
<tr>
<td>2.127</td>
<td>2.143</td>
<td>55.000</td>
</tr>
<tr>
<td>2.485</td>
<td>3.399</td>
<td>50.000</td>
</tr>
<tr>
<td>4.304</td>
<td>5.521</td>
<td>45.000</td>
</tr>
<tr>
<td>5.231</td>
<td>4.787</td>
<td>40.000</td>
</tr>
<tr>
<td>14.755</td>
<td>14.388</td>
<td>25.000</td>
</tr>
<tr>
<td>23.198</td>
<td>22.400</td>
<td>20.000</td>
</tr>
</tbody>
</table>

HEAT FLUX AT ROOT = 36229.72 BTU/HR-SQFT

n = 25
EPS = 0.20000
ACC = 0.0000000
CASE VI SAMPLE OUTPUT

### CASE VI PROBLEM

**FINITE LENGTH FIN WITH INSULATED END**
- **FIN LENGTH** = 6.000 FT
- **REACT TEMP.** = 1000.00°F
- **T INFINITY** = 140.00°F
- **EMITTANCE** = 0.400
- **FIN HALF THICKNESS** = 0.3333 FT
- **CONDUCTIVITY** = 170.00
- **EXponent** = 0.0000

<table>
<thead>
<tr>
<th>Location (X)</th>
<th>Temperature (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000 FT</td>
<td>100.00 deg. R</td>
</tr>
<tr>
<td>0.339 FT</td>
<td>100.00 deg. R</td>
</tr>
<tr>
<td>0.666 FT</td>
<td>100.00 deg. R</td>
</tr>
<tr>
<td>0.999 FT</td>
<td>100.00 deg. R</td>
</tr>
<tr>
<td>1.333 FT</td>
<td>100.00 deg. R</td>
</tr>
<tr>
<td>1.666 FT</td>
<td>100.00 deg. R</td>
</tr>
<tr>
<td>2.000 FT</td>
<td>100.00 deg. R</td>
</tr>
<tr>
<td>2.333 FT</td>
<td>100.00 deg. R</td>
</tr>
<tr>
<td>2.666 FT</td>
<td>100.00 deg. R</td>
</tr>
<tr>
<td>3.000 FT</td>
<td>100.00 deg. R</td>
</tr>
<tr>
<td>3.333 FT</td>
<td>100.00 deg. R</td>
</tr>
<tr>
<td>3.666 FT</td>
<td>100.00 deg. R</td>
</tr>
<tr>
<td>4.000 FT</td>
<td>100.00 deg. R</td>
</tr>
<tr>
<td>4.333 FT</td>
<td>100.00 deg. R</td>
</tr>
<tr>
<td>4.666 FT</td>
<td>100.00 deg. R</td>
</tr>
<tr>
<td>5.000 FT</td>
<td>100.00 deg. R</td>
</tr>
<tr>
<td>5.333 FT</td>
<td>100.00 deg. R</td>
</tr>
<tr>
<td>5.666 FT</td>
<td>100.00 deg. R</td>
</tr>
<tr>
<td>6.000 FT</td>
<td>100.00 deg. R</td>
</tr>
</tbody>
</table>

**HEAT FLUX AT ROOT** = 35844.32 BTU/HR-SQFT

**N** = 25

**EPS=0.0950610**

**ACC=1.001660**
CASE VIIa Sample Output

Finite Length Fin with Both End Temperatures Specified

Fin Length = 15,000 FT
Rect Temp. = 1000.00 R
Temp. at Outer End = 300.00 DEG. R
T Infinity = 500.00 R
Emittance = 0.40
Fin Half Thickness = 0.0208333 FT
Conductivity = 10.00
Exponent = 0.0000

x = 0.000 FT  t = 1000.00 DEG.R
x = 7,160 FT  t = 954.00 DEG.R
x = 6,866 FT  t = 934.00 DEG.R
x = 6,082 FT  t = 880.00 DEG.R
x = 1,204 FT  t = 764.00 DEG.R
x = 1,643 FT  t = 722.00 DEG.R
x = 2,353 FT  t = 650.00 DEG.R
x = 3,123 FT  t = 613.00 DEG.R
x = 4,394 FT  t = 550.00 DEG.R
x = 7,954 FT  t = 500.00 DEG.R
x = 11,763 FT  t = 450.00 DEG.R
x = 13,976 FT  t = 400.00 DEG.R
x = 14,172 FT  t = 350.00 DEG.R
x = 15,000 FT  t = 300.00 DEG.R

Heat Flux at Rect = 32722.12 BTU/HR-SQFT
Inflection Point Occurs at 7,954 FT
Heat Flux at Inflection Point = 1095.39 BTU/HR-SQFT
Heat Flux at Outer End = 6747.94 BTU/HR-SQFT

N = 25
Eps = .007021
Acc = 0.001000

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TABLE XXVII
CASE VII SAMPLE OUTPUT

CASE 7B PROBLEM
TEMP AT END OF PIN EQUALS T INF
FINITE LENGTH PIN WITH BOTH END TEMPERATURES SPECIFIED
FIN LENGTH = 15.0000 FT
ROOT TEMP = 1000.00 deg. R
TEMP. AT OUTER END = 500.00 deg. R
T INFINITY = 500.00 deg. R
EMITTANCE = 0.400
FIN HALF THICKNESS = 0.0208333 FT
CONDUCTIVITY = 100.00
EXponent = 0.0

<table>
<thead>
<tr>
<th>X (FT)</th>
<th>T (deg. R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1000.00</td>
</tr>
<tr>
<td>0.165</td>
<td>950.00</td>
</tr>
<tr>
<td>0.358</td>
<td>900.00</td>
</tr>
<tr>
<td>0.586</td>
<td>850.00</td>
</tr>
<tr>
<td>0.863</td>
<td>800.00</td>
</tr>
<tr>
<td>1.206</td>
<td>750.00</td>
</tr>
<tr>
<td>1.647</td>
<td>700.00</td>
</tr>
<tr>
<td>2.244</td>
<td>650.00</td>
</tr>
<tr>
<td>3.126</td>
<td>600.00</td>
</tr>
<tr>
<td>4.719</td>
<td>550.00</td>
</tr>
<tr>
<td>15.061</td>
<td>500.00</td>
</tr>
</tbody>
</table>

HEAT FLUX AT ROOT = 32703.72 BTU/HR-SQFT
FLUX AT OUTER END = 60.13 BTU/HR-SQFT

N = 25
EPS = 0.0000100
ACO = 0.0010000

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CASE 7C PROBLEM

FINITE LENGTH FIN WITH BOTH END TEMPERATURES SPECIFIED
FIN LENGTH = 15,000 FT
RECT TEMP. = 1000.00 DEG. R
TEMPS AT OUTER END = 200.00 DEG. R
T INFINITY = 160.00 DEG. R
EMITTANCE = 0.400
FIN HALF THICKNESS = 0.0208333 FT
CONDUCTIVITY = 100.00
EXponent = 0.0000

\[
\begin{align*}
X & = 0.000 \text{ FT} \quad T = 1000.00 \text{ DEG. R} \\
X & = 0.147 \text{ FT} \quad T = 950.00 \text{ DEG. R} \\
X & = 0.315 \text{ FT} \quad T = 900.00 \text{ DEG. R} \\
X & = 0.500 \text{ FT} \quad T = 850.00 \text{ DEG. R} \\
X & = 0.731 \text{ FT} \quad T = 800.00 \text{ DEG. R} \\
X & = 0.992 \text{ FT} \quad T = 750.00 \text{ DEG. R} \\
X & = 1.301 \text{ FT} \quad T = 700.00 \text{ DEG. R} \\
X & = 1.671 \text{ FT} \quad T = 650.00 \text{ DEG. R} \\
X & = 2.118 \text{ FT} \quad T = 600.00 \text{ DEG. R} \\
X & = 2.669 \text{ FT} \quad T = 550.00 \text{ DEG. R} \\
X & = 3.360 \text{ FT} \quad T = 500.00 \text{ DEG. R} \\
X & = 4.243 \text{ FT} \quad T = 450.00 \text{ DEG. R} \\
X & = 5.397 \text{ FT} \quad T = 400.00 \text{ DEG. R} \\
X & = 6.935 \text{ FT} \quad T = 350.00 \text{ DEG. R} \\
X & = 9.003 \text{ FT} \quad T = 300.00 \text{ DEG. R} \\
X & = 11.716 \text{ FT} \quad T = 250.00 \text{ DEG. R} \\
X & = 15.000 \text{ FT} \quad T = 200.00 \text{ DEG. R}
\end{align*}
\]

HEAT FLUX AT RECT = 34256.7C BTU/HR-SQFT
FLUX AT OUTER END = 1437.65 BTU/HR-SQFT

N = 25
EPS = 0.000000
ACC = 0.000000

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<table>
<thead>
<tr>
<th>Case 7C Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finite length fin with both end temperatures specified</td>
</tr>
<tr>
<td>Fin length = 15,000 ft</td>
</tr>
<tr>
<td>Rect temp = 1000.00 R</td>
</tr>
<tr>
<td>Temp. at outer end = 650.00 deg. R</td>
</tr>
<tr>
<td>T infinity = 500.00 R</td>
</tr>
<tr>
<td>Emissance = 0.400</td>
</tr>
<tr>
<td>Fin half thickness = 0.9205333 ft</td>
</tr>
<tr>
<td>Conductivity = 100.00</td>
</tr>
<tr>
<td>Exponent = 6.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Minimum temp. of 516.07 R occurs at 9.039 ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>0.000</td>
</tr>
<tr>
<td>0.165</td>
</tr>
<tr>
<td>0.335</td>
</tr>
<tr>
<td>0.504</td>
</tr>
<tr>
<td>0.663</td>
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<td>1.207</td>
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<td>1.649</td>
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<td>2.247</td>
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<td>3.139</td>
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<td>4.774</td>
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<tr>
<td>9.039</td>
</tr>
<tr>
<td>13.339</td>
</tr>
<tr>
<td>14.569</td>
</tr>
</tbody>
</table>

Heat flux at RCCL = 32697.67 BTU/HR-SCFT
Flux at outer end = -4432.69 BTU/HR-SCFT

N = 25
EPS = 0.00000000001
ACC = 0.00000001
CASE & PROBLEM

FINITE LENGTH FIN WITH RADIATING END
FIN LENGTH = 2.000 FT.
RUCT TEMP. = 530.00 R
T INFINITY = 160.00 R
EMITTANCE = 0.95
FIN HALF THICKNESS = 0.0416666 FT
CONDUCTIVITY = 29.37
EXPONENT = 6.1980

TEMPERATURE AT OUTER END = 483.48 DEG. R
X = 0.000 FT. T = 530.00 DEG. R
X = 0.803 FT. T = 500.00 DEG. R
X = 2.001 FT. T = 483.48 DEG. R

HEAT FLUX AT RUCT = 4903.88 BTU/HR-SCFT
FLX AT OUTER END = 97.42 BTU/HR-SCFT

N = 25
EPS = 0.0000010
ACC = 0.0010000
APPENDIX B

THERMAL CONDUCTIVITY OF ALUMINUM

In 1958, Lucks and Deem (5) reported values for the thermal properties of several engineering materials as a function of temperatures. One of these materials was 2024-T4 aluminum which had been heated to 600° F for one hour and then air cooled before testing.

The thermal conductivity of this material varies considerably with temperature but the variation may be approximated by the equation

\[ k = 29.37 \, (T)^{0.1980} \]

In this equation \( T \) has units of degrees Rankine and \( k \) has units of btu/hr sq ft R. This equation will predict the thermal conductivity of the material within 2% over the temperature range 210°R to 860°R. Tabulated values of actual and calculated values of the thermal conductivity are given in Table XXXI.

**TABLE XXXI**

<table>
<thead>
<tr>
<th>Temperature (°R)</th>
<th>True Conductivity (btu/hr sq ft R)</th>
<th>Calculated Conductivity (btu/hr sq ft R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>210</td>
<td>64</td>
<td>85</td>
</tr>
<tr>
<td>260</td>
<td>88</td>
<td>88</td>
</tr>
<tr>
<td>360</td>
<td>95</td>
<td>94</td>
</tr>
<tr>
<td>528</td>
<td>103</td>
<td>102</td>
</tr>
<tr>
<td>660</td>
<td>107</td>
<td>106</td>
</tr>
<tr>
<td>860</td>
<td>110</td>
<td>112</td>
</tr>
</tbody>
</table>

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Investigation of Constraints in Thermal Similitude, Volume I

Final Report

Paul L. Miller
Francis W. Holm

December 1969

This document has been approved for public release and sale; its distribution is unlimited.

The studies described in this report clarify the effects of some of the limitations imposed by the laws of thermal similitude, and determine the thermal modeling laws for a heat pipe.

In Volume 1 solutions were presented for the steady-state temperature distribution and heat transfer in a radiating fin having temperature dependent thermal conductivity. Using these solutions, modeling prediction errors were determined for fin type prototype/model systems with dimensional distortions, with material having temperature dependent thermal conductivity, and with low prototype temperatures. These prediction discrepancies ranged from very small errors to errors in heat transfer rate as high as 75% in a severely distorted model.

In Volume 2 the thermal modeling laws for a heat pipe were derived and experimentally verified. It was observed that prototype thermal behavior could be predicted, from model data, to within 30 F over the temperature range tested (140 to 330 F). Heat pipe failure due to capillary failure was also predictable to within 210 F.

A flexible heat pipe was also designed and experimentally tested. Performance was not degraded under conditions of bending.
<table>
<thead>
<tr>
<th>KEY WORDS</th>
<th>LINK A</th>
<th>LINK B</th>
<th>LINK C</th>
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