A SUGGESTED CRITERION FOR PILOTED HEADING CONTROL

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NOMENCLATURE

- $g$: Acceleration due to gravity
- $L$: Aerodynamic roll moment divided by roll moment of inertia
- $L_\lambda$: \( \partial L / \partial \lambda \) where $\lambda = \delta_w$, $\psi$, or $\beta$
- $L_\lambda^2$: \( (1 - \frac{1}{L_{\lambda}^2}) \)
- $M$: Aerodynamic yawing moment divided by yaw moment of inertia
- $N_\lambda$: $\partial N / \partial \lambda$ where $\lambda = \delta_w$, $\delta_r$, $\delta$, $r$, or $p$
- $N_\lambda^2$: \( (1 - \frac{1}{N_{\lambda}^2}) \)
- $N_0^2$: Numerator of 1/8 transfer function
- $p$: Roll rate
- $r$: Yaw rate
- $s$: Laplace operator
- $U_0$: Steady-state velocity
- $V_{CF}$: Alleron-to-rudder crossfeed function
- $V_0$: See Equation 6
- $V_{\delta\alpha}$: $\dot{\alpha} / \alpha$
- $\beta$: Sideslip angle
- $\Delta \alpha_{\max}$: Maximum sideslip excursion at the c.g. occurring within two seconds or one-half period of the dutch roll, whichever is greater, for a step alleron control command (see Ref. 1)
- $\delta_r$: Rudder pedal deflection at the cockpit
- $\delta_r(3)$: Rudder pedal deflection 3 seconds after a unit step lateral wheel input
- $\delta_r^*(3)$: Normalized rudder deflection; $\delta_r^*(3) = N_{\delta\alpha}\delta_r(3)$
- $\delta_w$: Lateral wheel (or stick) deflection at the cockpit

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\[ \Delta \]
Characteristic determinant-denominator for transfer functions

\[ \zeta_d \]
Dutch roll damping

\[ \mu \]
Crossfeed shaping parameter (See Equation 10)

\[ \phi \]
Bank angle

\[ \frac{\phi_{/\phi}}{A} \]
Roll/sideslip ratio in the dutch roll mode

\[ \phi_{osc/avg} \]
A measure of the ratio of the oscillatory component of bank angle to the average component of bank angle following a rudder-pedals free impulse aileron control command (see Ref. 1)

\[ \gamma_p \]
Phase angle expressed as a lag for a cosine representation of the dutch roll oscillation in sideslip (Ref. 1)

\[ \omega_d \]
Dutch roll frequency

INTRODUCTION

The ability to make precise changes in aircraft heading is a key factor in pilot evaluation of lateral-directional handling qualities. Assuming other good qualities (e.g., adequate roll response, yaw frequency/damping, etc., per Ref. 1), deficiencies in heading control, which can nevertheless exist, are directly traceable to excitation of the dutch roll mode due to roll-yaw cross-coupling effects. It is commonly accepted piloting technique to reduce these excursions by appropriate use of the aileron and rudder, usually referred to as "coordinating the turn." The problem is that existing criteria (see, for instance, Refs. 1-4) for heading control are based on aileron-only parameters, and the effects of rudder control are only indirectly apparent as they may have influenced individual pilot ratings. The fact that these criteria are not satisfactory is shown in Ref. 5, where several configurations which violated boundaries based on aileron-only parameters were given good to excellent pilot ratings. The approach taken here is that for an otherwise acceptable airplane the aileron-rudder shaping necessary to coordinate the turn is a dominant factor in pilot evaluation of heading control. In this regard it is important to recognize that heading control is basically an outer loop and cannot be satisfactory if the inner bank angle loop is unsatisfactory. Table 1 contains a set of requirements intended to serve as a checklist for good roll control. These requirements are discussed in some detail in Ref. 6.

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TABLE 1

GROUNDED RULES FOR APPLICATION OR RATING DATA TO ASSESSING CONTROL CRITERIA

1) \( T_R < 1.25 \)
2) \( \mu_R > 0.4 \)
3) \( I_d > 0.09 \) and \( \zeta_{aw} > 0.15 \)
4) \( |\alpha/\beta|_d < 1.5 \) when turbulence is a factor and \( |N_{aw}/Q_{aw}| > 0.03 \)
5) Meets \( L_d \) vs. \( \mu_R \) boundaries when \( |N_{aw}/Q_{aw}| \leq 0.03 \)
6) Meets Level 2 \( Q_{aw}/Q_{aw} \) in MIL-F-8705B
7) Pilot comments do not indicate:
   a) Significant roll control problems
   b) Control power or sensitivity problems
   c) Nonlinear control system problems such as friction, breakout, etc.
   d) Excessive gust response

ANALYSIS AND BASIC CONCEPT

In general, coordinated flight implies minimum yaw coupling due to roll entries and exits which can be quantified in many ways, e.g.: 1) zero sideslip angle \( (\phi = 0) \); 2) zero lateral acceleration at the c.g.; 3) turn rate consistent with bank angle and speed \( (r = \gamma/\psi) \); and 4) zero lateral acceleration at the cockpit (ball in the middle).

Conditions 1-3 are equivalent when the side force due to \( V_{aw} \) and \( I_d \) are very small, which is usually the case. The fourth turn coordination criterion is complicated by pilot location effects which, however, appear to be mainly associated with side qualities and not with heading control itself (Ref. 5). Based on these considerations it appears that sideslip angle is an appropriate indicator of turn coordination. Accordingly, the following formulation undertakes to identify the parameters that govern the aileron-rudder shaping required to maintain coordinated flight as defined by zero sideslip angle \( (\phi = 0) \).
With an aileron-rudder crossfeed, $Y_{CF}$, the rudder by definition, is given by

$$\delta_r = Y_{CF} \delta_w$$

(1)

For the assumed ideal (zero sideslip) coordination

$$\rho = \left(\frac{\delta_w}{\delta_r} + Y_{CF} \frac{\delta_r}{\delta_w}\right) \delta_w = 0$$

(2)

whereby the ideal crossfeed is

$$Y_{CF} = \frac{\delta_r}{\delta_w} = -\frac{\delta_w}{\delta_r}$$

(3)

For augmented airplanes, these numerators are high order and cannot be generalized. However, as was shown in Ref. 7, aircraft with complex augmentation systems represented by higher-order systems (HOS) tend to respond to pilot inputs in a fashion similar to conventional unaugmented aircraft or low-order systems (LOS). In fact, more recent unpublished work by the author of Ref. 7 showed that a HOS which cannot be fit to a LOS form is predictably unsatisfactory to the human pilot.

The appropriate LOS form for $Y_{CF}$ is based on the approximate factors for conventional airplanes obtained from Ref. 8 as follows.

$$Y_{CF} = \frac{N_{w,v}(s + \lambda_w(\epsilon/\omega))[(s + (1/\lambda_{y,f})]}{N_{w,v}(s + \lambda_w(\epsilon/\omega))[(s + (1/\lambda_{y,f})][s - (N_{c,x}/Y_{CF})]}$$

(4)

where

$$\lambda_i = \frac{\lambda_i - (\lambda_i - \lambda_i^*)^2}{\lambda_i^2 - (\lambda_i - \lambda_i^*)^2}$$

$$\lambda_i^* = \frac{1}{\lambda^*} = -\lambda_i + (\lambda_i - \lambda_i^*^2)(\lambda_i - (\epsilon/\omega))$$

(5)
For the frequency range of interest, i.e., excluding both low and high frequencies \( \alpha \ll s \ll \frac{\alpha}{\beta} \),

\[
Y_{CP} = -\frac{N_{CP}(s + 1/\tau_{CP})}{N_{DP}(s + 1/\tau_{DP})}
\]  

(5)

To provide a meaningful reference for the control crosscoupling term, \( N_{DP} \), in Eq. 5, it is expressed as the ratio of yawing to rolling acceleration, \( N_{DP}/\alpha_{r} \). Also, since the rudder sensitivity can be separately optimized and does not usually represent a basic airframe limitation, it is appropriate to remove it from consideration. Accordingly, the resulting LOS representation of the crossfeed, \( Y_{CP} \), is given as:

\[
Y_{CP} = C_{CP} \frac{N_{DP}}{\alpha_{r}} = -\frac{N_{CP} N_{DP}}{\alpha_{r}^{2}} \frac{N_{CP}(s + 1/\tau_{CP})}{N_{DP}(s + 1/\tau_{DP})}
\]  

(6)

Equation 6 indicates that the aileron-to-rudder shaping required to maintain coordinated flight \( (\beta = 0) \) is directly related to the separation between the aileron (wheel or stick) and rudder (pedal) sideslip zeros.

As a basis for direct correlation with pilot opinion, a "rudder mapping parameter," \( \mu \), is defined in "theoretical" form as

\[
\mu = \left( \frac{Y_{CP}}{N_{DP}} \right) - 1
\]  

(7)

The frequency response characteristics of \( Y_{CP} \), Eq. 6, as a function of the sign of \( \mu \) are shown in Fig. 1 in terms of literal expressions for the Bode asymptotes. These asymptotes indicate that the magnitude of the coordinating rudder is a function of \( N_{DP}/\alpha_{r} \) at all frequencies and that the shaping of the rudder response is determined by \( \mu \). These parameters are summarized in terms of their analytical and pilot-centered functions in Table 2.

*All derivatives are in the stability axis system.

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For $\mu > 0$

Lag Lead Compensation

$$|Y_{LF}| \frac{N_0}{L_{B_w}} \left[ 1 + \mu \right] \frac{1}{\frac{1}{T_R} \left( 1 + \mu \right) \frac{1}{s}}$$

$$\frac{N_0}{L_{B_w}} \frac{1}{T_R}$$

$\omega ---$

For $\mu < 0$

Lead Lag Compensation

$$|Y_{LF}| \frac{N_0}{L_{B_w}} \left[ 1 + \mu \right] \frac{1}{\frac{1}{T_R} |\mu|}$$

$$\frac{N_0}{L_{B_w}} \frac{1}{T_R}$$

Figure 1. Asymptotes of Aileron-Paddler Crossfeed
<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>ANALYTICAL FUNCTION</th>
<th>PILOT-CENTERED FUNCTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu )</td>
<td>Defines shape of ( Y_{CR} )</td>
<td>Determines complexity of rudder activity necessary for ideally coordinated turns. Also defines phasing of heading response when rudder is not used.</td>
</tr>
<tr>
<td>( N_{CR}/I_{CR} )</td>
<td>Defines magnitude of ( Y_{CR} )</td>
<td>Determines magnitude of rudder required and/or high-frequency yawing induced by aileron inputs.</td>
</tr>
</tbody>
</table>

The parameters \( N_{CR}/I_{CR} \) and \( \mu \) are a natural choice for correlation of heeling control pilot rating data since they completely define the aileron-to-rudder crossfeed necessary for turn coordination. Such an ideal crossfeed is difficult to isolate with simple flight test procedures, but is nevertheless considered a viable correlation concept because of modern usage (Ref. 1) which permits simulation and analysis methods to demonstrate specification compliance.

Since the rudder sequencing with aileron inputs is the key issue, it was decided to use a LOS form in the time domain. Assuming a unity high-frequency gain for the Eq. 6 form, and the ideal definition of \( \mu \) (Eq. 7), the rudder time history required to coordinate a unit step wheel or stick input is:

\[
\delta_R(t) = 1 + \mu(1 - e^{-t/N_{CR}}) 
\]

(8)

Note that \( \delta_R(t) \) refers to the rudder pedal motion (thereby including effects of rudder gearing and accounting for the SAS). Solving Eq. 8 for the rudder shaping parameter, \( \mu \):

\[
\mu = \frac{\delta_R(t) - 1}{1 - e^{-t/N_{CR}}} 
\]

(9)
The value of \( t \) used is properly set by the lower limit on the frequency range of interest for piloted heading control. The simulation experiments of Ref. 9 indicated that a minimum heading crossover of about 1/3 rad/sec was necessary for desirable handling qualities. Therefore, a corresponding time of 3 sec was selected as being most pertinent to a pilot-centered characterization of crossfeed properties. Recognizing further (Eq. i) that \( T_R \leq \frac{1}{\omega} \) is approximately equal to the roll mode time constant, \( T_R \), and that the latter must generally be less than 1.0 to 1.4 sec for acceptable roll control [Ref. 1] sets the following limits on the exponential in Eq. 9.

\[
T_R \leq 1.0^* \\
\leq 1.4^*
\]

\[
e^{-T_R/T_R} \leq 0.069 \\
\leq 0.117
\]

Accordingly, Eq. 9 reduces within a maximum error of 5-10 percent, depending on airplane class, to

\[
\mu = \delta_T(3) = 1
\]

(10)

This simple relationship was used to compute \( \mu \) for the pilot rating correlations later shown.

However, before this simple formula can be applied it is necessary to avoid the high-frequency responses which occur due to pairs of roots which frequently occur with complex SAS installations having associated higher-order \( \delta \) numerators. For example, a simple washout yaw rate feedback and a first-order lagged aileron rudder crossfeed results in seventh-order \( \delta \) numerators of unaugmented airplanes. Most of the zeros of these polynomials occur at very high frequency, having negligible effect on the dynamics near the pilot's crossover frequency, and therefore should not be accounted for in the shaping function \( \mu \). The standard procedure utilized to compute the values of \( \mu \) was to eliminate all roots of the \( \delta \) numerators above values of 6 rad/sec in pairs, i.e., keeping their order relative to each other the

*For small, light, or highly maneuverable airplanes.

†For medium to heavy weight, low to medium maneuverability airplanes.
same (e.g., a third over fourth would be reduced to a second over third order, etc.). Roots above 6 rad/sec which do not occur in pairs are left unmodified.

The following example illustrates a typical computation of $\mu$ and the effect of removing the high-frequency roots from Eq. 2. The aileron-rudder crossfeed for one of the Ref. 10 configurations used in the pilot rating correlations is given as:

$$\frac{\delta_r}{\delta_w} = \frac{.19(s + .102)(s + .222)}{(s + .057)(s + 5.6)(s + 109.9)}$$  \hspace{1cm} (11)

As discussed above, all roots above 6 rad/sec are removed in pairs and the high-frequency gain is set to unity, resulting in the following equation:

$$\frac{\delta_r}{\delta_w} = \frac{(s + .102)(s + .222)}{(s + .057)(s + 5.6)}$$  \hspace{1cm} (12)

The rudder time responses to a unit wheel input for Eqs. 11 and 12 are plotted in Fig. 2. Removal of the high-frequency roots is seen to replace

![Figure 2. Effect of Removing High-Frequency Roots from $\delta$ Numerators](image)

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the initial rapid rudder reversal with a unity initial condition. These responses are essentially equivalent to the pilot who sees the necessity to use immediate rudder with aileron inputs (which must be removed 1/sec later). The value of $\mu$ corresponding to this response is $\delta_r(3) - 1 = -1.7$.

Figure 3 presents typical coordinating ($\mu = 0$) rudder time histories for step aileron inputs on a grid of $\mu$ vs. $N_{\alpha W}/L_{\alpha W}$. Moving vertically on this grid changes the shape ($\mu$) of the crossfeed, $\gamma_{CP}$, keeping the initial value (high-frequency gain) constant. Moving horizontally produces a change in the crossfeed gain ($N_{\alpha W}/L_{\alpha W}$) at all frequencies without changing the shape. Note that this is consistent with Table 2 and Fig. 1, where it is shown that $\mu$ dictates the required aileron-to-rudder shaping and $N_{\alpha W}$ defines the magnitude of the gain for all times (and frequencies). The basic shapes of the time histories in Fig. 3 are indicative of the fundamental assumption that the rudder time history can be fit by the Fig. 6 form. The basic implication of this form is that the rudder response is essentially monotonic in the frequency range of interest.

For augmented airplanes the effective values of $N_{\alpha W}/L_{\alpha W}$ (which represent the high-frequency yawing and rolling accelerations) are taken as the high-frequency gain of the simplified $\alpha/L_{\alpha W}$ and $\gamma/L_{\alpha W}$ transfer functions, e.g., all roots above 6.0 rad/sec are taken as equivalent gains. In effect this defines $N_{\alpha W}$ and $L_{\alpha W}$ as the yawing and rolling accelerations due to a wheel (or stick) input at frequencies above 6.0 rad/sec. A physical interpretation relating the crosscoupling derivatives $N_{\alpha W}$ and $L_{\alpha W}$ with the rudder shaping parameter, $\mu$, is given in Table 3.

Known values of $N_{\alpha W}/L_{\alpha W}$ and $\mu$ define a unique aileron-to-rudder coordination time history as discussed above, so it is possible to establish how (or if) pilot rating of heading control is dominated by such coordination requirements by plotting applicable pilot rating data on a grid of $\mu$ vs. $N_{\alpha W}/L_{\alpha W}$.

A summary of the data sources considered is given in Table 4. Each of the data points found to be applicable to heading control (i.e., met the ground rules) is plotted and paired on a logarithmic grid of $N_{\alpha W}/L_{\alpha W}$ vs. $\mu$ in Fig. 4. Only in-flight and moving-base simulator data were considered.
Contrails

\[ \frac{N_{\delta w}}{L_{\delta w}} < 0 \]

\[ \delta_r \]

\[ t \]

\[ \mu > 0 \]

\[ \frac{N_{\delta w}}{L_{\delta w}} > 0 \]

\[ \delta_r \]

\[ t \]

\[ \mu = 0 \] (\( \delta_r \) is constant)

\[ 0 \]

\[ \frac{N_{\delta w}}{L_{\delta w}} \] increasing \( \rightarrow \)

\[ \delta_r \]

\[ t \]

\[ 0 > \mu > -1 \]

\[ \mu = -1 \] (steady state value is zero)

\[ -1 \]

\[ \frac{N_{\delta w}}{L_{\delta w}} \] increasing \( \rightarrow \)

\[ \delta_r \]

\[ t \]

\[ -1 > \mu > -2 \]

\[ \mu = -2 \] (steady state value is minus initial value)

\[ -2 \]

\[ \frac{N_{\delta w}}{L_{\delta w}} \] increasing \( \rightarrow \)

\[ \delta_r \]

\[ t \]

\[ -2 > \mu \]

step aileron input at \( t = 0 \)

+ rudder is into the turn

Figure 5. Typical Rudder Time Histories for Zero Sideslip

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### Table 3. Physical Interpretation of μ

| μ > 0 | \( N_{\mu \gamma} \) and \( N_{\alpha \delta} \) are additive, indicating that the crosscoupling effects increase with time after an aileron input. |
| μ = 0 | \( N_{\mu} \) = \( g/\alpha \), indicating that all roll-yaw crosscoupling is due to \( N_{\mu \gamma} \). The aileron-rudder crossfeed is therefore a pure gain. |
| \(-1 < \mu < 0\) | \( N_{\mu \gamma} \) and \( N_{\alpha \delta} \) are opposing. Initial crosscoupling induced by \( N_{\mu \gamma} \) is reduced by \( N_{\alpha \delta} \) as the roll rate builds up. Exact cancellation takes place when \( \mu = -1 \), resulting in a zero rudder requirement for steady rolling. |
| μ << -1 | Low-frequency and high-frequency crosscoupling effects are of opposite sign, indicating a need for complex rudder reversals for coordination. If rudder not used, the nose will appear to oscillate during turn entry and exit. |

### Table 4. Summary of Current Data

<table>
<thead>
<tr>
<th>TYPE OF AIRCRAFT SIMULATED</th>
<th>DESCRIPTION OF SIMULATOR</th>
<th>REFERENCE</th>
<th>TOTAL NUMBER OF DATA POINTS</th>
<th>NUMBER OF POINTS MEETING GROUND RULES</th>
</tr>
</thead>
<tbody>
<tr>
<td>Executive jet and military Class II</td>
<td>Variable stability T33</td>
<td>11</td>
<td>84</td>
<td>16</td>
</tr>
<tr>
<td>STOL</td>
<td>Variable stability helicopter</td>
<td>10</td>
<td>109</td>
<td>30</td>
</tr>
<tr>
<td>General aviation (light aircraft)</td>
<td>Variable stability Navion</td>
<td>12</td>
<td>26</td>
<td>6</td>
</tr>
<tr>
<td>Jet fighter-carryer approach</td>
<td>Variable stability Navion</td>
<td>13</td>
<td>36</td>
<td>22</td>
</tr>
<tr>
<td>Space shuttle vehicle</td>
<td>6 DOF moving base with Redifon display (NASA Ames FSD)</td>
<td>5</td>
<td>52</td>
<td>52</td>
</tr>
<tr>
<td>STOL</td>
<td>3 DOF moving base (NASA Ames S-16)</td>
<td>2</td>
<td>8</td>
<td>7</td>
</tr>
</tbody>
</table>

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Figure 4. Pilot Rating Correlation with Crossfeed Parameters
With the exception of one or two points the data from all the sources in Table 4 coalesce quite nicely. The criterion in Fig. 4 is conservative in that the few points that do not fit are rated better than the other data in the same region.

**Physical Interpretation**

The iso-opinion lines in Fig. 4 indicate that some values of the rudder shaping parameter, \( \mu \), are more desirable than others in that they are less sensitive to an increase in aileron yaw. The following observations help to explain this trend in terms of pilot-centered considerations:

1) Moderately high proverse (positive) \( N^p_{\mu} \) is acceptable in the region where \( \mu \approx -1 \). Physically, this corresponds to a sudden initial heading response in the direction of turn followed by decreasing rudder requirements. (Required steady-state rudder is zero when \( \mu = -1 \), see Fig. 3.) It is felt that the pilots are accepting the initial proverse yaw as a heading lead and are not attempting to use cross control rudder.

2) The allowable values of proverse \( N^p_{\mu} \) decrease rapidly as \( \mu \) becomes greater than \(-1\). Physically, this corresponds to an increase in the requirement for low-frequency cross control rudder activity (see Fig. 3), which is highly objectionable.

3) The pilot ratings are less sensitive to the required rudder shaping when \( N^p_{\mu} \) is negative (adverse yaw). Recall that adverse yaw is consistent with conventional piloting technique.

It is significant that the pilot rating correlations are not dependent on the type of aircraft and in fact are shown to be valid for vehicles ranging from light aircraft to fighters, STOL, and space shuttle configurations. This result indicates that good heading control characteristics are dependent on a fundamental aspect of piloting technique (aileron-rudder coordination) and that such factors as aircraft size, weight, approach speed, etc., can be neglected for all practical purposes. It is felt that the invariance of ratings with aircraft configuration is related to the pilot's ability to adapt to different situations and to rate accordingly. Finally, the excellent

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correlations of pilot ratings with the aileron-rudder crossfeed characteristics indicates that the required rudder coordination is indeed a dominant factor in pilot evaluation of heading control.

The rudder shaping parameter is attractive as a heading control criterion because the handling quality boundaries are easily interpreted in terms of pilot-centered considerations.

$N_{0w}'/U_0$ NEAR ZERO

Control crosstown effects are obviously not a factor when $|N_{0w}'/U_0|$ is small. This may occur when the basic control crosscoupling is negligible or with augmentation systems which result in ideal crossfeeds, $\gamma_{CF}$, having denominators of higher-order dynamics than numerators (e.g., the augmented $N_{0w}$ is zero). For $N_{0w}'/U_0$ identically zero, the required aileron-rudder crossfeed takes the Bode asymptotic form shown in Fig. 5 for unaugmented conventional airplanes. The rudder magnitude required to coordinate mid-frequency and high-frequency aileron (wheel) inputs is seen to be dependent on the roll crosscoupling, $g/U_0 = N_p'$, whereas low-frequency rudder requirements are dependent on $N_p$. The required rudder shaping has the characteristics of a rate system (ramp input) at low and high frequency.

![Diagram of crossfeed](image)

Figure 5. Required Crossfeed for $N_{0w}' = 0$
Accordingly, aileron-rudder shaping per se is not the essence of the problem, which reduces, instead, to concern with the general magnitude of the required rudder crossfeed. Utilizing the same response considerations as in the computation of $\mu$, $b_\theta(3)$ is suggested as the correlating parameter when $|N_{\theta\mu}/I_{\theta\psi}|$ is small or when the denominator of $Y_{CP}$ is of higher order than the numerator. In order to remove rudder sensitivity effects, which can be separately optimized, $b_\psi(3) = N_{\theta\psi}/b_\theta(3)$ is used as a correlating parameter. The question of what specifically constitutes a "small" value of $N_{\theta\psi}/I_{\theta\psi}$ has proven to be somewhat difficult to quantify. Reasonably good correlations were found by plotting the $N_{\theta\psi}/I_{\theta\psi} < 0.05$ pilot ratings vs. $b_\psi(3)$ as shown in Fig. 6. More recent experience in utilizing the $\mu$ parameter has revealed several unacceptable configurations, where $N_{\theta\psi}/I_{\theta\psi}$ is slightly greater than 0.05, that fall within the acceptable region in Fig. 4. Plotting these configurations on the $b_\psi(3)$ criterion revealed the deficiency in all cases tried.

Based on this experience, the rules for application of the $\mu$ parameter have been revised as follows. If $|N_{\theta\psi}/I_{\theta\psi}| < 0.07$, plot $\mu$ vs. $N_{\theta\psi}/I_{\theta\psi}$ on the Fig. 4 criterion and $b_\psi(3)$ on the Fig. 6 criterion and utilize the most conservative result. Note that $b_\psi(3)$ is simply the $b_\theta (3)$ used in the $\mu$ calculation (Eq. 10) multiplied by $I_{\theta\psi}$ [recall that $Y_{CP} = Y_{CP}(N_{\theta\psi}/I_{\theta\psi})$ and that $b_\psi(3) = N_{\theta\psi}/b_\theta(3)$]. If $|N_{\theta\psi}/I_{\theta\psi}| > 0.07$ the Fig. 4 criterion is used without the need to check $b_\psi(3)$.

**COMPLEX Rudder Shaping**

As discussed earlier, reasonable fits of the HOS to the LOS form implicit in Eq. 10 presume that the aileron-to-rudder shaping is at least monotonic in the region of piloted control. This assumes that if the required rudder coordination to a step aileron input is non-monotonic in the region of control, pilot opinion will be poor. Since we have been unable to find any configurations which have such a non-monotonic shape, and which have been tested for pilot opinion of heading control, it is not possible to quantify the mismatch effects at this time.
## Table: Data Source vs \( \delta_d \)

<table>
<thead>
<tr>
<th>Sym</th>
<th>Data Source</th>
<th>( \delta_d )</th>
</tr>
</thead>
<tbody>
<tr>
<td>□</td>
<td>Ref. 10</td>
<td>0.2</td>
</tr>
<tr>
<td>□</td>
<td>Ref. 5</td>
<td>0.2</td>
</tr>
<tr>
<td>□</td>
<td>Ref. 5</td>
<td>0.4</td>
</tr>
<tr>
<td>□</td>
<td>Ref. 12</td>
<td>0.1</td>
</tr>
<tr>
<td>□</td>
<td>Ref. 13</td>
<td>0.1 - 2</td>
</tr>
<tr>
<td>□</td>
<td>Ref. 13</td>
<td>0.4</td>
</tr>
<tr>
<td>□</td>
<td>Ref. 2</td>
<td>0.24 - 0.37</td>
</tr>
<tr>
<td>□</td>
<td>Ref. 11</td>
<td>0.1 - 2</td>
</tr>
<tr>
<td>□</td>
<td>Ref. 11</td>
<td>0.34</td>
</tr>
</tbody>
</table>

### Diagram

- **Shaded Points**: \( \omega_d \leq 1.0 \)
- **Open Points**: \( 1.0 < \omega_d \leq 2.0 \)
- **Flagged Points**: \( \omega_d > 2.0 \)

**Extrapolation**: \( \delta_d = -1.15 \)

**Figure 6. Pilot Rating Correlations When \( |N_{\text{yaw}}/N_{\text{yaw}}| \) Is Small**
CONCLUSIONS

The main conclusions are summarized as follows:

1) Pilot evaluation of heading control is highly correlate-able with the aileron-rudder sequencing required to coordinate turns.

2) Very good correlation has been obtained with data from widely varying configurations.

The results to date are very encouraging. The crossfeed parameter seems to have potential as a heading control criterion. Additional experimental data to investigate certain regions of the criterion planes in Figs. 4 and 6 and to investigate the effects of complex non-monotonic required rudder shaping would be desirable.


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APPENDIX

In the Calspan critique (Ref. 14), two example configurations are used to question the viability of the $\mu$ parameter. This appendix is presented not only as a rebuttal, but also to illustrate some examples of proper application of the parameter.

The first example is configuration PE from Ref. 15. It was specifically chosen (in Ref. 14) because the pilot ratings and flight test commentary indicated severe deficiencies in heading control (average rating = 6 and maximum rating = 10, with three pilots), yet it plots inside the Level 2 boundary on the current $\mathcal{H}E/K$ specification. The configuration was plotted on the $\mu$ criterion boundaries of Fig. 4 and shown to fall within the Level 1 region. Unfortunately, it was plotted incorrectly in that $H_{MW}/L_{MW}$ was not converted to stability axis. More important, however, is the fact that for low values of control crosscoupling such as for Configuration PE, the $\delta_k(3)$ parameter should also be calculated. Since $\delta_k(3)$ is simply $\delta_T(3) \times L_{MW}$, it can be obtained directly from Ref. 14, Fig. 3 (which plots $\delta_T$ vs. time) as $-1.7 \times 1.04 = -1.77$ ($L_{MW} = 1.04$ was obtained from Ref. 15). From Fig. 6 it can be seen that a $\delta_k(3)$ of -1.77 represents such extreme adverse yaw that it does not even fit on the plot! However, if we extend the linear extrapolation, a predicted rating of 8.5 results, which is in excellent agreement with the pilot ratings. It should be noted that in the strictest sense the version of $\mu$ published in Ref. 6 indicated that $\delta_k(3)$ applied when $|H_{\delta V}/L_{\delta V}|$ is less than 0.03, whereas a value of 0.05 was utilized in Ref. 14. However, it should have been realized that even a slight reduction in airspeed, such as would occur when maneuvering, would reduce $H_{\delta V}/L_{\delta V}$ to values less than 0.03. This fact, plus the repeated pilot commentary that yaw coupling due to roll rate [which is what $\delta_k(3)$ is all about] was extremely objectionable should have made it obvious that the $\delta_k(3)$ parameter should at least be checked. Since the publication of Ref. 6 we have increased the applicable range of $\delta_k(3)$ from $|H_{\delta V}/L_{\delta V}| > 0.05$ to 0.07 with the provision that both $\mu$ and $\delta_k(3)$ should be checked and the worst case utilized. It is hoped that this will remove the necessity to apply engineering judgment in the selection of the appropriate criterion.

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A benefit from the Ref. 14 critique is that we now have a data point in a region where no data were previously available. It is gratifying that this new point supports the extrapolated boundary shown in Fig. 6.

There is continued implication throughout the Ref. 14 critique that a first-order model must be identified before the μ criterion may be applied. This is erroneous. As stated in this paper and as was stated in Ref. 6, high-frequency roots (at or above 6 rad/sec) which occur in pairs (denominator and numerator) should be removed from \( Y_{C1} \). No other modification is required. A simplified crossfeed which also removes the low-frequency roots is used in Fig. 4 of Ref. 14 to calculate \( \tilde{\delta}_\theta(3) \). This is nonstandard usage which if corrected would modify the (Ref. 14) Fig. 4 time history; accordingly, the Ref. 14 criticism that the simplified (Fig. 3, Ref. 14) crossfeed provides nearly perfect coordination only out to 7 sec would be modified somewhat. However, as stated in the μ development paper, abrupt alleron inputs longer than 3 sec are of no interest for closed-loop heading control. We would therefore consider the β time history shown in Fig. 4, Ref. 14 representative of excellent coordination for the precision heading control task.

The second example cited by Calpan (an early version of the YF-16) also turned out to be quite beneficial in terms of lending additional insight into application of the μ parameter. In this case the required rudder to coordinate was found to be extremely non-monotonic (Fig. 6, Ref. 14). The general nature of the shape of the actual required rudder and the shape suggested by the lower-order equivalent system defined by μ is shown in Fig. 7. The extreme mismatch between the L0S and LOS precludes even a cursory evaluation of μ. However, the complex nature of the required rudder to coordinate a step alleron input would in itself lead one to suspect very poor pilot opinion of heading control. While Calpan was not able to produce a pilot rating for this configuration, it is well known that the original version of the YF-16 was an extremely poor airplane (pilot ratings of 9 and 10). As mentioned in the paper, no data are available to date to allow us to extend the longitudinal results of Ref. 7 to the lateral-directional axis, e.g., that the existence of a very poor LOS match is in itself a measure of poor handling. These data, rather than invalidating μ, actually provide the first available data which tend to support the assumed extension of the Ref. 7 results.
As noted by Calspan in their presentation, some of the $\phi_B$ values used by STI in Ref. 6 were in error. This issue was covered in correspondence between Calspan and STI over a year ago. However, the tendency to miscalculate $\phi_B$ is perhaps an inherent deficiency in the parameter. Witness the discrepancies for the same flight conditions between the values of $\phi_B$ which appear in two separate Calspan authored reports:

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Reference 4</th>
<th>Reference 11</th>
</tr>
</thead>
<tbody>
<tr>
<td>2P2</td>
<td>-295</td>
<td>-234</td>
</tr>
<tr>
<td>3N0</td>
<td>-189</td>
<td>-224</td>
</tr>
<tr>
<td>3N2</td>
<td>-344</td>
<td>-290</td>
</tr>
<tr>
<td>4P2</td>
<td>-532</td>
<td>-208</td>
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<tr>
<td>12A2</td>
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<tr>
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<td>-210</td>
<td>-167</td>
</tr>
<tr>
<td>12P2</td>
<td>-356</td>
<td>-291</td>
</tr>
</tbody>
</table>

No attempt has been made to determine which of these represents the "correct" values of $\phi_B$. 

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Finally, Ref. 14 states that "crossfeed of aileron stick through a shaping network is an idealization that does not properly represent the task the pilot must accomplish." We could easily take issue with this statement by noting the lack of positive evidence (in Ref. 14) experimentally quantifying the pilot's rudder activity, e.g., in describing functional or other terms. However, in fact it does not even matter what internal feedback the pilot must use to generate the appropriate rudder; what does matter is the characteristic shape and magnitude of the rudder required to coordinate stick inputs no matter how the pilot chooses to generate it. If the magnitude is large, or the shape complex, he will not like it. In fact, he may not use the rudders at all, in which case the complex shaping or large magnitude required will show up as a lack of consonance between bank angle and yaw rate.

QUESTIONS AND ANSWERS

1. Ralph Smith: Is there a difference in short time tracking performance from adverse to positive yaw at the corresponding Level 1 boundaries?

   Yes. Short term tracking performance was best for $\alpha = \alpha_1$ and proves $B_{0w}/B_{0y}$. Physically the heading response leads the bank angle in this region which tends to give abrupt lateral motion at the cockpit but results in very tight tracking.

2. Chick Chalk: How is $\delta_2(3)$ normalized when $\frac{B_{0w}}{B_{0y}} < 0.05$.

   $\delta_2(3) = \frac{B_{0w}}{B_{0y}}$. Implication is that $B_{0y}$ is removed and can be separately optimized. Also note that $\delta_2(3)$ is calculated any time $\frac{B_{0w}}{B_{0y}} < 0.07$.

3. Dwight Schaeffer, Boeing: What happens if a lot of $\beta$ is needed to coordinate turn?

   Coordinated turn is defined as when yaw and roll are in consonance, e.g., $r = \frac{\beta \sin q}{V}$. We have assumed that this is well approximated when $\beta$ and $\dot{\beta}$ are zero, viz.,

   $$(s - Y_r)\dot{\beta} + r + \frac{\beta \sin q}{V} = Y_{0y} - Y_y$$

   Underlying assumption is $Y_{0y} = Y_y \neq 0$.
b. Jerry Lockemour, Northrop: If would be instructive to show \( \beta \)-coordination for aileron pulse instead of a step because \( \beta \) (croop) transient upon input removal is important. Also for depressed reticle bomb sights sideslip upon aileron removal can be helpful in minimizing the "pendulum effects." Any comment? Did you consider \( I_{\alpha} \)?

a. Our philosophy in showing an aileron step is that it is illustrative of the pilot's rudder action required to initiate a bank. Once the bank is established, the removal of aileron is considered to be a step in the opposite direction.

b. The use of sideslip to quicken the heading response shows up in the \( \mu \) parameter as a "bulge" allowing large proverse \( \theta_b/\dot{L}_v \) at \( \mu = -1 \).

c. \( I_{\alpha} \) is implicit in the \( \beta \) numerators. The answer is therefore, yes.