The work presented in this report was completed by Mr. J. D. Ingram under Contract No. AF 33(617)-10945, Project No. 41377, Task No. 417757 for the Air Force Flight Dynamics Laboratory, Research and Technology Division, Air Force Systems Command. The work was undertaken from May 1963 through June 1964, under the supervision of Dr. A. C. Bringen and Dr. J. C. Samuels.

This report has been reviewed and is approved.

[Signature]
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ABSTRACT

A representative survey of the literature of Aerodynamic Noise was made. This was not intended to be an exhaustive list but rather to give the state of the field presently. It was found that much of the research done in this field is either experimental or semi-empirical. This is primarily due to the complex nature of the theory of turbulence.
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1. Nature of the Problem

When a vehicle moves through air there are two basic means by which it can produce noise: (1) by its propulsion mechanism (motor-jet, rocket, etc.,) and (2) by its interaction with its surroundings. At low speeds the first of these is by far the dominant one while near or above the speed of sound, mechanism (2) becomes quite important.

The reason for the study of the noise produced by such a mechanism is likewise two-fold. First, the noise field is one of the important reasons for failure in vehicles such as rockets and high speed jets. Secondly, human beings find objectionable the "sonorous sequence of whines, roars, and bangs" which are the sounds produced by jet aircraft. It is from one of these two standpoints that one must view most of the research done on aerodynamic sound. The first is primarily a near field study while the second is far field in nature.

Since most of the high speed propulsion devices of today involve the impingement of a jet exhaust into relatively quiescent air this will constitute the area of study for the previously mentioned propulsion devices. We will exclude the effects of jet air temperature and intermittent combustion, etc.

In the second category of noise production one finds primarily the pressure field due to a turbulent boundary layer. Such phenomena as the sound produced by the interaction of a vortex with a shock wave also must be included in this category.

From the point of view of definition we may define sound produced aerodynamically as all sound fields which owe their existence to airflows rather than being by-products of the vibrations of solids. It has been found valuable, however, by Lightall and others to distinguish between sound and "Pseudo Sound". Inside an airflow pressure fluctuations commonly occur due to instability which yields at low Reynolds numbers a regular eddy pattern and at high Reynolds numbers an irregular turbulent flow. If a microphone or other pick up device is placed in the flow it will record the density fluctuations due to such patterns just as if they were an ordinary sound field. However, they lack the essential quality of sound, i.e. propagation at or around the velocity of sound. Hence, the designation "pseudo sound". This does not imply that such a flow does not generate a propagating
sound field but rather that the density fluctuations in the flow itself do not constitute such a sound field. This is a fact often overlooked by experimenters in the field.

The problem of the near field of aerodynamic noise where the fluctuations due to the sound field and those due to the pseudo sound field are comparable (they fall off with inverse distance and inverse square distance) is very hard to treat theoretically. It is here, however, that the strongest implications for the structural engineer are found. Much of the progress in this area has been of an experimental nature.

2. The Basic Mechanisms of Aerodynamic Sound Generation

We may recall that in the classical theory of acoustics, a simple source is one which caused a fluctuation in the net flow through a closed surface. An example is a sphere executing radial vibrations. A dipole source results from the pulsating motion of a rigid body in an acoustic medium. An example of this would be the pressure field of a vibrating string which vibrates in only a single plane. It is not necessary, however, that a rigid body be moving and the acoustic medium be at rest. In fact if the converse is true an acoustic field is also produced. As an example of this, consider the fluctuating drag force on a cylinder due to flow perpendicular to its axis. Even though the cylinder remains stationary the reactive forces set up an acoustic field in the moving fluid quite analogous to that which would result from a stationary medium and a vibrating cylinder. This accounts for the theory of edgetones such as the sound from the wind through telephone wires.

There is, in the example just given, an implicit assumption which underlies almost all of the theoretical studies on flow noise, i.e. that the pressure field may be obtained by taking the forces resulting from flow as forcing functions on an acoustic medium. This ignores the interaction of the sound field itself with the unstable flow, an effect which is important at very small Mach number but which is almost completely insignificant in the study of aircraft noise.

The basis for the theoretical understanding of sound produced aerodynamically was given by Lighthill [1]. Combining the equations of motion and mass conservation he obtained the equation
\[ \frac{\partial^2 \rho}{\partial t^2} - \frac{1}{a^2} \frac{\partial^2 \rho}{\partial x^2} = \frac{\partial^2 \rho_{ij}}{\partial x_i \partial x_j} \]

where

\[ \rho_{ij} = \rho \nabla \cdot V + P_{ij} - \frac{\partial}{\partial x_i} p \delta_{ij} \]

\( \rho \) is the density, \( V \) the velocity, \( P \) the ordinary stress tensor, and \( a \) the velocity of sound. This equation yields directly a solution for the acoustic problem if one has sufficient information about the turbulence stress tensor \( \rho_{ij} \). The determination of the structure of the turbulence generated by airflow is then the fundamental problem for the determination of flow noise. This turns out to be, however, a virtually insurmountable obstacle to a complete theoretical treatment. Only for isolated cases has any attempt been made to analyze completely the noise produced by a given type of turbulent flow. Instead of attempting to solve this problem researchers have exchanged it for the determination of equivalent moving "source" distributions. It is supposed that the noise from the flow can be synthesized by a set of simple or multipole sources distributed throughout the flow. The character of these source distributions and the qualitative estimates which can be obtained from application of the exact theory has occupied most of the theoretical and experimental work on jet and rocket noise.

3. Jet Noise

It has been previously mentioned that the two basic types of aerodynamic noise generators are: (1) noise from jets impinging on quiescent air and, (2) noise induced by turbulence around the skin of the aircraft. The first of these presents several peculiar problems. Since the method to be applied is that of finding source distributions within the flow which are equivalent to the forces the flow would exert on an acoustic medium we must consider reflection and refraction of the acoustic field on the velocity discontinuity which defines the boundary of the jet. This is a problem inherent to the method of sources. Then we must consider the fact that these are not stationary sources but are being convected with the flow. It is found that both of these considerations have profound effects on the directionality and power from the flow. They account for the heart shaped pattern of the pressure isochar.
The radiation fields of a moving source, dipole, and quadrupole respectively are
\[
P_{0} = \frac{m(t-a/a_{0})}{4\pi r(l-M \cos \theta)} \frac{x_{1} P_{1}(t-r/a_{0})}{4\pi r^{2}(l-M \cos \theta)} \frac{x_{1} P_{1}^{*}(t-r/a_{0})}{4\pi r^{2}(l-M \cos \theta)}
\]
where the functions \(m(t)\), \(P_{1}(t)\), and \(P_{1}^{*}(t)\) are their strengths at time \(t\). \(K_{c}\) is the Mach number of convection. \(r\) is the distance from the point of emission, and \(\theta\) is the angle between the direction of emission and the direction of convection.

From dipoles and quadrupoles, whose radiation fields depend on incomplete cancelling of signals from positive and negative sources due to differences in time of emission, additional factors \([l-M \cos \theta]^{-1}\) appear, representing the increase in those time differences. This is closely related to the Doppler increase in the observed frequency.

The method of using moving-source fields in practical problems of jet noise has been perfected by Proven Williams [2-4]. The experimental evidence for jets is not quite as convincing as for boundary layer turbulence that the problem at hand is a convected pattern of turbulence. However, it seems safe to regard the jet case as a moving system of quadrupoles.

4. Boundary layer Noise

The noise generated by turbulence in boundary layers is an important type of noise of the dipole type. Probably in no other area of aerodynamic noise has there been experimental exploration having a similar quality to that for boundary layer turbulence. It has been shown that wall pressure fluctuations are much larger than skin-friction fluctuations and hence in the main the acoustic dipoles are perpendicular to the surface.

From the measurements of Willmarth [5,6] we may infer that the autocorrelation function of pressure fluctuations is roughly Gaussian, and that the area under the correlation curve is given with good accuracy by
\[
\int_{0}^{\infty} P_{\omega}(\omega) P_{\omega}(\omega+\gamma) d\gamma = \frac{1.5\sigma^{2}}{u} \text{p}^{2}
\]
where \(\sigma\) is the displacement thickness. Thus we can regard pressures at a point as well correlated within a time separation of about \(1.5\sigma^{2}/U\) and uncorrelated for greater time separations.

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Willmarth also finds that a relatively slowly changing pressure pattern is moving down stream so fast that the time variation at a single point gives an impression of high frequency that is misleading. This is rather an indication that the random space pattern possess a high speed of convection.

The measurements mentioned above provide an excellent base for the studies for the engineer who wishes to examine structural fatigue and the random character of boundary layer noise. However, without an adequate theory for a proper interpretation these experiments do not attain their full significance for the understanding of this noise mechanism.

It is readily seen that both for jet noise and for boundary layer noise the fundamental problem is the nature of the turbulence involved. The entire theoretical effort now being expended on the area seems to be aimed at devising means of circumventing this block without understanding the physical nature of turbulence. It would appear very unlikely that any great theoretical advances can be made in the general area of aerodynamic noise until a significantly new and fertile theory of turbulence appears.
Summaries From Aerodynamic Noise

The motivation for most of the studies of noise produced by airflows which were done prior to 1952 was the understanding of the eddy patterns set up by steady flows. These studies resulted from observations of such flows as the acoustic noise of wind through telephone wires. It is seen from the work of Kutta and Joukowski that the force \( F \) which acts on a cylinder of length \( b \) is

\[
F = \rho \pi D \frac{U}{2} b
\]

where \( \rho \) is the fluid density, \( \Gamma \) the circulation about the cylinder, and \( \frac{U}{2} \) is the velocity of the flow. Hence when the eddies of \( -2\Gamma \) and \( +2\Gamma \) are thrown off alternately from the flow these results an alternating force on the cylinder normal to the direction of flow.

The acoustic dipole, discussed by Rayleigh and Lamb is generated by the fore and aft displacement of a cylinder or sphere in a fluid. The similarities between this and the situation of the eddies from a cylinder was the motivation for Yudin [7] to suppose that the origin of vortex noise lies in the variable force acting on the medium during the flow past a body. He inferred that the pressure on the cylinder must be dependent on the strength of the eddies in the wake. From a dimensional analysis he deduced that the sound power would vary as the sixth power of the wind velocity. Kakiniseev [8] extended Yudin's supposition. The ultimate statement of these works is that the far acoustic field produced by flow around fixed, rigid bodies is the same as if the fluid force sustained by the body were applied in the opposite direction to a corresponding fluid at rest. This idea also forms the core of the analysis by Etkin, Koechler and Iseke [9] in which it is supposed that the cylinder can be replaced by a body force sufficient to prevent the fluid's motion. This work then forms the basis for the proposition of Lighthill's theory of aerodynamic noise, part I of which appeared in 1952.

In part I of a classic paper Lighthill [1] proposed the fundamental basis of noise produced by gas flows with rigid boundaries. Most of the work done by earlier investigators of noise produced by flows was to determine the frequency content of sound produced by regular eddy patterns. Lighthill concerns himself with the determination of the intensity from an arbitrary flow. The method of attack is to estimate the flow field from aerodynamic principles and then to deduce the sound field. This precludes any interaction of the sound with the flow producing it.
The moving fluid is superposed on an appropriate acoustic medium at rest and the difference of the conservation of momentum equations is considered to be a fluctuating force field (known if the flow is known). This force field acts on the stationary acoustic medium and hence radiates sound according to the ordinary laws of acoustics. The difference between the stresses in the real flow and the fluctuating pressure in the acoustic medium are taken to be

\[ \tau_{ij} = \rho V_i V_j + P_{ij} - \frac{\partial}{\partial t} \delta_{ij} \rho \]

where \( V_i \), \( P_{ij} \), \( \rho \), and \( \delta_{ij} \) are the velocity, the real stress tensor, the density, and the acoustic velocity. \( \delta_{ij} \) is the Kronecker delta. The principal generator of acoustic disturbance is the Reynolds' stress since the dissipation of acoustic energy by viscosity and heat conduction is slow and likewise the differences \( p - \rho \delta \) is slight. It is noted that since much of the fluctuation of the momentum flux is balanced by a local reciprocating motion the effect is as if we were dealing with an acoustic quadrupole. This is a manifestation of the surface nature of the stress, and is a very inefficient method of converting kinetic energy to acoustic energy. The source density decomposes into a fluctuating dipole source

\[ \frac{1}{\lambda_0^2} \frac{\partial^2}{\partial t^2} \tau_{ij} \]

(where the summation convention is used on the indices) and a field of lateral quadrupoles due to the fluctuating shearing stresses.

At distances large compared with the dimensions of the flow the density fluctuations are given by

\[ \rho - \rho_0 = \frac{1}{\lambda_0^2} \frac{x_1 x_2 x_3}{x} \int \frac{1}{2} \frac{\partial^2}{\partial t^2} \tau_{ij}(y_1 y_2 y_3 - \delta_{ij} \delta) \] \( \delta \)

where the repeated indices indicate summation over the range 1, 2, 3. Here \( y_1 \) is the location of the source point and \( y_2 \) is the point at which the density fluctuation is measured.

Defining the intensity of acoustic radiation at a point by

\[ \frac{\rho_0^2}{\rho} \]

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where \( \sigma^2 \) is the mean square fluctuation in density at that point, we obtain for the intensity

\[
I(x) = \frac{1}{2\pi^2 \rho_0^2} \int \int \frac{(x_1 - x_2)(x_3 - x_4)(x_5 - x_6)}{|x-x_1|^3 |x-x_2|^3} \ dp \cdot \text{d}x
\]

\[
\frac{2}{\sigma^2} \int \left( \frac{\partial^2}{\partial t^2} \tau_{ij} \right) \left( \frac{\partial^2}{\partial t^2} \tau_{lm} \right) \ dx \ dy
\]

where a bar over a quantity indicates averaging in time.

For the acoustic power we have

\[
R = \frac{1}{60 \beta \rho \sigma^2} \int \frac{\partial^2}{\partial t^2} \tau_{ij} \left( x_t - \frac{1}{\sigma} \right) \ dx \ dy
\]

By physical arguments and dimensional analysis Lighthill concludes that at a distance \( x \) from the center of the flow the density fluctuations are roughly proportional to

\[
\rho_0 \left( \frac{x}{a_0} \right)^{\frac{3}{2}} \frac{x}{a_0}
\]

where \( u \) and \( s \) are a typical velocity and a typical length respectively. The intensities are predicted to be roughly proportional to

\[
\rho_0 u a_0 \frac{3}{2} \frac{s}{a_0}
\]

In Lighthill [10] applied the theory developed in part I to the scattering of sound by turbulence. He estimates that the energy scattered per unit time from a unit volume of turbulence is

\[
\frac{\sigma^2}{\alpha^2} \frac{1}{\lambda^2} \frac{\text{r}^2}{2}
\]
where $I$ is the intensity, $\lambda$ the wavelength of the incident sound and $a$ is the speed of sound. $\frac{1}{2}$ is the mean square velocity and $\lambda$ the macro-scale of the turbulence in the direction of the incident sound. This formula does not assume any particular kind of turbulence but does assume that $\lambda/a$ is less than one. It is predicted that components of the turbulence with wave number $k$ will scatter sound of wave number $k$ at an angle of $2 \sin^{-1}(k/2\lambda)$. The statistics of successive scatterings is considered and it is predicted that sound of wavelength less than the macro-scale of the turbulence $\lambda$ will become quite random in its directional distribution in a distance approximately $\lambda^2/\nu^2$. The theory is extended to include an incident acoustic pulse and incident shock waves. In the case of the shock it is necessary to take into account the actual speed of the shock relative to the fluid behind it. This predicts a value of $0.3\lambda$ times the kinetic energy of the turbulence traversed by a weak shock of strength $s$. The energy thus freely scattered when turbulence is convected through the stationary shock wave pattern in a supersonic jet may form an important part of the sound field of the jet.

Although broadly based on well-established principles, the details of Lighthill's theory involved new techniques developed to deal with certain aspects of the aerodynamic noise problem. The need to relate the quadrupole source strength to the details of the turbulence was a severe demand on the new theory. Although Lighthill succeeded in giving the quadrupole distribution of sources in terms of the structure of the turbulence the infancy of the theory of turbulence itself limits the applicability of this development. At low speeds Lighthill showed that under certain conditions the fluctuating Reynolds stress can be used as a good approximation for the stress tensor. On this basis Pronasov [18] computed the noise radiated by decaying isotropic turbulence. This is probably the only quantitative application of Lighthill's theory feasible with turbulence in its present state.

Pronasov assumes that the Reynolds number of the isotropic turbulence is large and its Mach number small. He indicated that the noise appears to be generated mainly by those eddies whose contribution to the dissipation of kinetic energy by viscosity is negligible. It is shown that the intensity of
sound at large distances is the same as would result from a
volume distribution of simple acoustic sources occupying the
turbulent region. The local value of the acoustic power out-
put $P$ per mass of turbulent fluid is given approximately by
the formula

$$P = \frac{1}{2} \rho c \frac{\partial \bar{u}}{\partial x} \left( \frac{\bar{u}^2}{\bar{u}} \right)^{5/2}$$

where $\alpha$ is a numerical constant, $\bar{u}$ is the mean square velocity
fluctuation, and $c$ is the velocity of sound. The numerical
value $\alpha$ is obtained by assuming Eisenberg's theoretical spectrum
of isotropic turbulence. It is found that the effects of decay
do not contribute greatly to the value of $\alpha$.

In [11] Lighthill published part II of his theory of
aerodynamic sound. This treats turbulence as a source of
sound. The theory is developed with special reference to the
noise of jets for which a detailed comparison with experiment
is made. The quadrupole distribution of part I is shown to
behave as if it were concentrated in independent point quad-
upoles, one in each "average eddy volume." The sound field
of each one of these is distorted in favor of downstream
emission by the general downstream motion of the eddy. This
explains for jet noise the marked preference for downstream
emission and its increase with jet velocity. Although tur-
bulence without any mean flow can produce noise as shown by
Prandtl, the intensity is much enhanced by a prevalent shear.
This sound has a directional maximum at $45^\circ$ to the direction
of the shear flow. The acoustic efficiency of the jet is of
the order of magnitude of $10^{-10}$ or $10^9$ where $M$ is the orifice
Mach number. A consideration of whether terms in the stress
other than the fluctuating momentum flux might become impor-
tant in heated jets indicates that they should hardly ever be
dominant. However, it should be emphasized that whenever
there is a fluctuating force between the flow and solid
boundaries dipole sources arise which may be more efficient
than the quadrupole radiation, at least at low Mach numbers.

Nilsen [12] treated the problem of interaction of turbulence
with a shock wave. He found that the presence of a shock wave
provides a mechanism for the transference of energy between the
modes of vorticity, entropy and sound. Formulas for spectra
and correlations have been found and numerical calculations
carried out to yield curves of the root mean square velocity
components, temperature, and noise in db against Mach number.
for the range $1 < M < \infty$. Both isotropic and anisotropic
initial turbulence have been considered.

Phillips [13, 14, 15] and Curle [16] have considered an
extension of the Lighthill theory to include the effect of
solid boundaries. This is a natural and important effect
since particularly at low Mach numbers the quadrupole nature
of the sound produced by a purely aerodynamic source is quite
inefficient. The influence of rigid boundaries is twofold:
(1) reflection and diffraction of the sound waves, and (2) a
resultant surface dipole distribution at the solid bound-
daries which are the limits of Lighthill’s quadrupole
distribution. It is shown that these effects are exactly
equivalent to a distribution of dipoles each representing the
force with which unit area of the solid boundary acts on the
fluid. A dimensional analysis shows that the intensity of
sound generated by the dipoles should at large distances $x$
be of the general form $p U^2 \rho \frac{1}{x^5} \cos \frac{\pi x}{L} \frac{\sin \frac{\pi x}{L}}{\frac{\pi x}{L}}$.

The theory developed by Curle [16] has been applied by
Phillips [15,17] and Kraichnan [18-22] to the generation of
sound by turbulence over an infinite flat plate. The generation
of noise by the interaction of the turbulence with the solid
boundaries has been named by Phillips as the aerodynamic surface
sound. If the motion of the fluid over the plate is statistically
homogeneous in planes, parallel to the plate then, he concludes,
the radiated dipole sound vanishes. This is said to be a
consequence of the vanishing or the mean square momentum per
unit area of the shear layer. However, when the motion is not
homogeneous in this fashion a non-zero acoustic radiation field
may be set up. Therefore, it is suggested that a semi-infinite
flat plate placed in a uniform stream would produce a finite dipole
radiation per unit area which becomes vanishingly small with increas-
ing distance down the plate.

Most of the experimental work to this time on jet aircraft
had centered on the engine itself as the primary source of noise.
However, the theoretical work of Lighthill and others who followed
indicated that boundary layer noise from turbulence would contribute
to in-flight noise of the aircraft. In this light, experimental
work on noise from turbulent shear flows began to appear. Coles
[23] and Harrison [24,25] made measurements in the boundary layer
on a smooth flat plate in super sonic flow. In situ measurements
of the aerodynamic noise and boundary layer profile for an air-
plane wing were made by Hull and Aligranti [26] and McI nod and
Jordan [27]. These indicated that the velocity profiles of the

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boundary layer were similar to the typical experimental turbulent boundary-layer velocity profiles. The ratio of the root-mean-square sound pressure on the surface to the free stream dynamic pressure was found to decrease linearly with increasing Mach number up to about $M = 0.55$ and remain constant from there up to the limiting Mach number of the airplane. A sharp increase in the sound pressure near the limiting Mach number of the aircraft was attributed to local shock formation on the wing.

It can be seen that if the turbulent boundary layer around an aircraft's fuselage is an important source of sound for vehicles in flight then this will also be a significant mechanism for the production of noise inside the hull. In this relation Ribner [28] and Cores [29] have explored the noise induced in the interior of jet aircraft by a turbulent boundary noise. The noise produced here is more profound than that studied theoretically previously since the shell of the airplane may act as a sound board and vibrate itself. It is assumed that the fluctuating pressure distribution induces ripples on the skin of the aircraft. The acoustic effects of such ripples in an infinite sheet are examined. Supersonic moving ripples case strong sound in the form of Mach waves while subsonic ones radiate no sound. Formulas for the mean square surface pressure and energy flux are obtained for an idealized turbulent pressure spectrum. The results are adapted to a practical fuselage.

In Proudman's [10] analysis of noise from isotropic turbulence the evaluation of the numerical constant $\alpha$ was made based on the assumption of Heisenberg's theoretical spectrum. Moller and Hatchet [30] and Newari [31] have extended their results to show the dependence of $\alpha$ on time. The authors infer from their computations that reduction in eddy size may have an important effect on the noise produced by turbulence. H. Z. Ribner (App. Mech. Rev. 5202) disagrees with this on the basis that the eighth power dependence on turbulent velocity far outweighs the weak effect of eddy size.

A great deal of the work done on aerodynamic noise with and without coupling to solid boundaries is accounted for by English and Canadian scientists. The work in Canada in this area is reviewed by Naka and Ribner [28]. The specific theoretical and experimental investigations described include: Axial flow; boundary layer noise (rigid and flexible wall); effects of boundary layer noise on aircraft structures; distribution of noise sources along a jet; ground run up mufflers; transmission of sound from, and acoustic energy flow in, a moving medium; sound generated by by interaction of a vortex with a shock wave.

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In a separate report Ribner [33] has discussed the distribution of noise sources along a jet. This is investigated by application of Lighthill's theory to regions of "similar" profiles. The analysis refers to the noise power emitted by a "slice" of the jet, (i.e. the section formed by two planes normal to the axis), as a function of distance x of the slice from the nozzle. It is found that this power is essentially constant with x in the initial mixing region (x² law), then further downstream (say 8 or 10 diameters from the nozzle) falls off extremely fast (x⁻⁷ law or faster) in the fully developed jet. Because of this striking attenuation of strength with distance it is concluded that the majority of noise by such a jet is generated in the mixing region and very little comes from the fully developed jet. A qualitative interpretation is given for Powell's experiments and the behavior of multiple nozzle or corrugated mufflers, both as to overall quieting and frequency shifting, is also interpreted in the light of the results.

Ribner [34] has also studied the flow of acoustic energy in a moving medium. Comparison of three authors (Blondinet [8], Johnson and Iaporte [36], and Ribner [25]), is made for similarities and discrepancies and an attempt is made to infer a correct formulation. He attempts to show by means of examples how variations in the velocity of a stream carrying acoustic energy in the form of plane waves can change the "linear theory" acoustic energy density from positive through zero to negative, with corresponding changes in the energy flow.

The acoustic radiation from isotropic turbulence has also been studied by Neches and Ford [37]. Their aim is to study the features of this sound which is independent of the manner in which it is produced. Through the use of the Navier-Stokes equation Lighthill's expression for the acoustic radiation is expressed in terms of a single variable for very large Reynolds' numbers. A new Lagrangian type of correlation is defined in such a way that a similarity argument can be applied without difficulty from the convective effects. Such convective effects then only enter through negligible Doppler shifts. Using this similarity results and Lighthill's formulation the self noise power spectrum for the turbulence is found. This spectrum at the high frequency end is proportional to \( \omega^{-7/2} \) (i.e. \( \omega > a \bar{V}/L \)) where \( \omega \) is the acoustic angular frequency, \( \bar{V} \) for the turbulence, \( L \) for the turbulent eddies, and \( a \).
is the velocity of sound. At the high frequency and this
spectrum is also independent of the diverging mechanism. Further-
more, the spectrum is proportional to \( w^2 \) at the low frequencies
and depends on the large scale eddies there.

Lighthill's theory of aerodynamic sound is for noise produced
by flow without coupling to solid boundaries. While this is
fundamental to an understanding of aerodynamic noise, practical
considerations demand attention to the effect of the coupling
between the solid boundaries of the flow and the noise produced
by it. In connection with this the output of a sound source in
reflecting environments was studied by Waterhouse [36]. He obtains
the output of simple point and dipole sources as functions of source
position by the method of images and a theorem due to Rayleigh on
the output of groups of simple sources. The cases of a dipole
source near a reflecting plane and a simple source near a reflecting
edge and corner are treated, and the effect of band width of the
source is considered.

In an application of this technique Waterhouse has studied the
sound power output by subsonic jets by a reverberation chamber
technique. Velocity profiles with round and square distributions
were studied in conjunction with nozzles of various shapes. The
frequency limitation of the reverberation chamber technique was
found to be 20 kHz due to absorption by the air.

In Lighthill's theory jet noise is generated by fluctuating
Reynolds stresses \( \tau \) (quadrupoles) within the jet. Much of
the work done theoretically in the area of aerodynamic noise has
centered on the determination of the source strengths (dipole
and quadrupole) which the Reynolds' stress is equivalent to.
However, since the extremely difficult problem of determining
the nature of the turbulent jet has been exchanged for the much
simpler one of finding equivalent distributions of sources
these inherent difficulties of the problem appear elsewhere. For
example, once one envisions the sources as being in the interior
of the jet, the radiation which eventually reaches the relatively
stationary medium has been altered by the convective motion of
the sources and reflection and refraction at the boundaries of
the jet.

Cheng [39,40,41] has attempted to treat this problem by
hypothesizing "image" calls of quadrupoles on the surface of the
jet. He seeks to relate the location and directional radiation
of these images to those in the interior by an estimate of plane
wave refraction. It is true that there may exist a distribution

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of sources in the quiescent air which will provide the same
acoustic field as the jet. However, it seems unlikely that
these are related in any simple fashion to the interior
quadropoles inasmuch as determination of acoustic images by
refraction through a curved surface is fallacious.

Pakan and Hall [48] studied the problem of the effect of
forward velocity on sound pressure for a whole jet engine in
flight by means of a moving microphone on the aircraft. They
found no detectable difference caused by increase in speed of
the airplane.

The noise of a turbulent jet was studied by Powell [43-48].
He considers a similarity argument for jets of varying dimensions.
Using the idealized estimate that each slice of the cross section
of the jet emits a single frequency he found a low frequency spectrum
dependent on $U_D^2$ where the high frequency part depends on $U_D^2 f^2$,
where $U_D$ is a typical turbulent velocity, $D$ is the diameter of the
jet, and $f$ is the frequency of the turbulence. The overall noise
level still depends on $U_D^2 f^2$. There is a strong indication that
the peak of the noise spectrum should emanate from the region close
to the end of the potential core, probably from a little downstream
of it. This analysis is very similar to that applied by H. S. Rhiner
[33]. Below is reproduced a table given by Powell which related
similarity theory and experiment.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Similarity theory</th>
<th>Experiment</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise power</td>
<td>$(\rho_u)^{\alpha}$</td>
<td>$\epsilon = 0$</td>
<td>a, b</td>
</tr>
<tr>
<td></td>
<td>$\beta = 2$</td>
<td>$\alpha = 8$</td>
<td>d, e, a, b</td>
</tr>
<tr>
<td>Peak frequency</td>
<td>$\frac{U_D^2}{\beta}$</td>
<td>$\alpha = 1$</td>
<td>f, g, h</td>
</tr>
<tr>
<td></td>
<td>$\beta = 1$</td>
<td>$\alpha = 0.5$</td>
<td>e</td>
</tr>
<tr>
<td>High frequency</td>
<td>$(\rho_u)^{\alpha}$</td>
<td>$7.9 &lt; \alpha &lt; 10$</td>
<td>e, h</td>
</tr>
<tr>
<td>spectrum</td>
<td>$\beta = 4$</td>
<td>$0.5 &lt; \beta &lt; 1$</td>
<td>e, h</td>
</tr>
<tr>
<td></td>
<td>$\beta = 1$</td>
<td>$4 &lt; \alpha &lt; 2$</td>
<td>e, a, b, h</td>
</tr>
<tr>
<td>Low frequency</td>
<td>$(\rho_u)^{\alpha}$</td>
<td>$5.6 &lt; \alpha &lt; 7.6$</td>
<td>d, e, g</td>
</tr>
<tr>
<td>spectrum</td>
<td>$\beta = 5$</td>
<td>$1 &lt; \beta &lt; 5$</td>
<td>d</td>
</tr>
<tr>
<td></td>
<td>$\beta = 1$</td>
<td>$0 &lt; \alpha &lt; 2$</td>
<td>d, e, g</td>
</tr>
</tbody>
</table>

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It is seen that much of the difficulty of applying Lighthill's theory lies in the conceptual difficulty of the acoustic quadrupoles $\rho U U$. Several authors, Nesham and Ford [66]; Corcos [49,50]; Ribner [31,32], have considered a theory based on the replacement of the quadrupoles by simple sources. The form of the equation of propagation is found to be

$$\frac{1}{2} \frac{D_x^2 P_{(1)}}{e_0} = P_{y(1)} = -\frac{1}{2} \frac{D_x^2 P_{(0)}}{e_0},$$

where

$$\frac{D}{dt} = \frac{\partial}{\partial t} + U \frac{\partial}{\partial x}, \quad \frac{D}{dt} = \sum_{i=1}^{3} \frac{\partial}{\partial x_i},$$

is the portion of the pressure containing all the compressibility effects, and $P_{(0)}$ is the portion of the pressure due to incompressible flow and satisfies the equation

$$\rho \frac{\partial^2 P_{(0)}}{\partial t \partial x_j} = \rho_0 \frac{\partial^2 P_{(0)}}{\partial x_j \partial t_j},$$

where $\rho_0$ is the ambient density. Thus the term $-\frac{1}{2} \frac{D_x^2 P_{(0)}}{e_0}$ represents a density of simple sources. The sources though individually non-directional represent the directional characteristics of jets through their distribution. Further directionality comes about by refraction of the sound field by the mean shear flow.

For unbounded low speed flows the equivalence of the simple source method and Lighthill's method have been examined by Ribner [33]. For bounded flows (i.e., taking into account the rigid surface causing the turbulence) the volume integral of simple sources still describes the radiation while to the
distributed quadrupoles one must append a surface integral of dipoles. Ribner states that similarity conditions for jets not only recover the $U^2$ law for total noise power but also the new law describing the distribution of noise energy with distance $x$ along the jet: these are reported to decrease as $x^0$ (constant) for the mixing region with a transition to $x^{-7}$ for the fully developed jet. Calculations for a simulated jet show how narrow frequency bands of the source spectrum appear greatly broadened by convection of the sources past the observer. Corresponding calculations for the radiated sound field automatically produce the correct Doppler shifted frequencies without an implicit introduction of the shift. This technique has been greatly employed and studied in the following years and does provide a simpler conceptual view of the noise. It does not, however, eliminate the fundamental difficulty in the study of aerodynamic noise, that of the structure of the turbulent flow.

Dyr [54,55] has studied the axial distribution of sources in a turbulent jet. It is shown that they can be found from: (1) the spectrum of total radiated power and (2) the frequency of the sources as a function of location along the jet axis. [Note that this information is experimentally available and throws light experimentally on the problem just mentioned.] With the use of existing data on the sound power spectrum and recently reported data on the frequency dependence of the most probable source location, an approximate axial distribution is derived.

Willmarth [5,6] and Favre et al [56] has studied the space-time correlations and wall pressure beneath a fluctuating boundary layer experimentally. It is found that the ratio of the root mean square wall pressure to the free stream dynamic pressure is a constant $\sqrt{\bar{P}^2/\bar{n}} = 0.06$ independent of Mach number and Reynolds number. In addition, space-time correlations in the stream direction show that pressure fluctuations whose scale is greater than 0.3 times the boundary layer thickness are convected with the convection speed $U = 0.62 U$, where $U$ is the free stream velocity, and have lost their identity in a distance approximately equal to ten boundary layer thicknesses.

In a related sense to the above experiments, Wilson [57] has studied experimentally the noise generated by turbulent flow around a rotating cylinder.
In an attempt to relate the far field acoustic characteristics to the flow variables for jets, such as rocket exhausts, which attain supersonic velocity, Howes [58,59] studied the similarity relations for such jets. In order to do this the jet was divided into two regions: (1) a region of mixed subsonic and supersonic flows and (2) a region of completely supersonic flow downstream. It has been shown experimentally that the subsonic region possesses mean flow characteristics, such as velocity profile, similar to those for a completely supersonic jet. The similarity relations specifically discussed are the total acoustic power level, power spectrum, directivity, local mean square pressure spectrum, and pressure probability density. These relations are first derived and then correlated with experimental data obtained from noise studies of various sizes of cold and hot air jets as well as turbojet engines. Good agreement between the predicted and experimental values of these parameters was obtained for subsonic flow and for all but the total acoustic power for supersonic flow. Portions of the results from this work is given in the section on experimental data.

In analyses of the sound field produced by turbulence the multipole source for the field is given by the fluctuating turbulent stress tensor $\tau_{ij}$. However, to a static observer, much of this fluctuation $I$ due to convection of a spatial flow pattern which changes slowly with respect to the time scale of the stress tensor. By analyzing the turbulent structure according to a frame of reference which moves with the turbulence, Frowes Williams [2] removed this fluctuation. The equations governing the acoustic output when written in this coordinate system are functions of the quadrupole strength alone. These equations show the important effect of convection on the acoustic field produced by turbulence. The total acoustic power emitted from a region is enhanced by convection and the radiation field no longer exhibits the characteristic symmetry about the quadrupole axes. The sound is favored for downstream emission. The basic result of this paper is that the intensity of acoustic radiation from a flow where the eddies convect through a finite volume but only exist within that volume depends on the Mach number according to the factor $(1-M \cos \theta)^{-2}(1+M \cos \theta)$, where $M$ is the Mach number of the eddies and $\theta$ that of the source volume.

In a second article Frowes Williams [3] has analyzed homogeneous convected turbulence by the same technique. Experimental techniques for measuring the velocity of convection are discussed.
and some properties of acoustic sources in convected turbulence are considered and particular reference is made to jet mixing regions.

Skudrzyk and Haddad [50] has attempted to predict the levels of boundary-layer noise and the noise produced by surface roughness as a function of the speed and the frequency on the basis of flow-noise studies with a rotating cylinder and measurements in the boundary layer. This work was applied to experimental results.

Powell [61-63] has considered once more the problem of aero-
dynamic noise and the plane boundary. It has been mentioned before that if one wishes to include the effects of solid boundaries into Lighthill's theory it is necessary to appeal to the response due to the distributed quadrupoles a surface integral over dipoles created by a force analogous to the Kothe-Joukovsky lift force. In certain situations an argument can be made to the effect that these dipole forces vanish ("reflection principle"). Powell states that image principle is developed in a rigorous manner and the apparent paradox is resolved with the help of an extension of Lighthill's and Curle's analyses to include boundaries which are not wholly immersed in the sound generating flow. Indeed it is reported that the pressures exerted on the plane boundaries are simply reflections of the quadrupole generators of the flow itself. Thus, the pressure dipoles account for an enhancement of the quadrupole power, in fact a quadrupling when the wave-length is large, except that degeneration into octupoles occurs for those lateral quadrupoles of the type that would be associated with fluctuations across the shear of an adjacent boundary layer. Under these circumstances it should be possible to estimate the noise of a plane turbulent boundary layer with satisfactory accuracy from sufficient knowledge of the principal quadrupole source strength alone.

Lighthill himself observed a paradox in the u' velocity dependence of the acoustic power coupled with the strongly directional character of jet noise. Powell [64] suggested that if the noise generators of turbulent jets undergo convection effects which are limited in such a way as to follow similarity behavior this paradox can be resolved directly. This hypothesis is said to be at least plausible to a first approximation owing to the general velocity field of the jet having typical dimensions comparable to a fraction of the wavelength. An important corollary is the expectation of appreciable refraction effects. Aspects relevant to the directional peaks of the higher frequencies, being less pronounced and located further from the jet axis, are discussed.
In connection with the refraction effects mentioned in the previous paper Gottlieb [65] has studied the sound field resulting from a source placed near a velocity discontinuity. Fourier representations of the source, reflected, and refracted waves are used. Applications of the method of stationary phase is used to obtain the far field. It is found that this far field is strongly peaked in some directions and reduced in others. These are illustrated graphically.

Rihner [55] and Powell [66] effect of a moving source on the resulting acoustical field. They reported that a high frequency source imbeded in a patch of moving fluid emits a constant acoustic power independent of the motion, although the directivity is altered. This holds when the wavelength is less than the radius of the moving region. At the other extreme of low frequencies it appears that the acoustic power is enhanced, somewhat as the emission of a source is enhanced by movement through a fluid at rest. Typical wavelength of a radiating eddy in a jet lies somewhere between the two extremes and a limited convective enhancement of the power is inferred. The amount should be less than that predicted by Lighthill; it could conceivably lie within experimental error accounting for the very close adherence to the $u^3$ law.

Dyer and Franken [67] Chobotek and Powell [68], and Eldred [69] have given an account of the general noise environment of flight vehicles. Amplitude and space-time correlations are given for noise due to the turbulent exhaust stream of a motor and it is shown that the noise associated with the turbulent wake of a flight vehicle is qualitatively similar to it but that the most intense radiation is directed up stream. Other noise sources considered to be of practical significance are boundary layer pressure fluctuations and shock oscillations.

Dock [70] has considered the acoustic radiation from a turbulent fluid containing foreign bodies of arbitrary shape. The problem is formally solved in terms of Green's functions. It is shown that is fluctuations in the fluid are locally isentropic the volume source distribution of the pressure fluctuations is quadrupole. A proof is given of the proposition that when arbitrary obstacles are immersed in a fluid all dipole radiation must come from surface distributions on these bodies. It is also proved that if the density fluctuations or normal density gradient fluctuations vanish on the surface then there is no dipole radiation. The total acoustic power radiated by
a turbulent boundary layer on an infinite rigid plane is estimated as a representative example of acoustic radiation from a turbulent boundary layer.

On the experimental side, Howes [71], Gerrard [72], Lauter [73], Lawrence [74], Lillie [75] and Mall [76,77] have studied pressure fluctuations on the boundary of a subsonic jet. A semi-empirical analysis of the equation for incompressible fluctuations in a turbulent fluid, using similarity relations for round subsonic jets with uniform exit velocity, is used to predict the shape of the time-averaged fluctuation-pressure distribution along the mean velocity boundary of jets. The predicted distribution is independent of the distance downstream of the nozzle exit along the mixing region, inversely proportional to the distance downstream along the region of mean velocity self-preservation, and proportional to the inverse square of the distance downstream along the fully developed region. The experimental data is in fair agreement with these predictions. However, the measured fluctuation-pressure distribution were found to be very sensitive to jet temperature and velocity profile, particularly in the vicinity of the nozzle.

Bies [78] has generalized the work of Ingard and Lamb [79] in investigating the effect of a reflecting plane on an arbitrarily oriented multipole. He finds that for a dipole a distance \( h \) above an infinite, rigid reflecting plane the ratio of the acoustic power radiated to that radiated in the absence of a plane is

\[
\frac{\Psi}{\Psi_0} = (1 + \frac{\Psi}{H} \left(\frac{\sin H}{H^2} - \frac{\cos H}{H}\right) \sin ^2 \theta)
\]

\[
+ \left(1 - \frac{\Psi}{H} \left(\frac{\sin H}{H^2} + \frac{2}{H} \cos H\right) \cos ^2 \theta \right)
\]

where \( \theta \) is the angle of the dipoles' axis with the normal to the plane and \( H = 2\pi h/\lambda \). Similar expressions are found for the quadripole. The large variations in radiated power observed for a single multipole tend to diminish for a distribution of such poles.

A jet or rocket engine and its small scale model are examples of dynamically similar systems if the engines are geometrically scaled and if average velocities, temperature, and densities are the same at similar locations. When the appropriate scaling relationship for pressure fluctuations has been established for
such similar systems, the model can be used as a tool for the investigation. Such relationships have been studied by Morgan and Sutherland [58,83]. It is found that pressure fluctuation amplitudes at similar positions are the same when measured in constant-percentage-frequency bands and when the frequency is scaled inversely proportional to a characteristic length.

Several of the previous references have treated the sound from an aerodynamic source as being generated by fluid dilations only and hence as a volume distribution of simple sources. Powell [82,85,86] states that physical reasoning suggests that aerodynamic sound from free flows may be attributed to local fluid dilations accompanied by local fluid accelerations, the former acting as a source field and the latter as a field of dipoles which reduce to quadrupoles. A supporting mathematical method is given. It discloses the formal need for an additional surface integral.

In the 1961 Iabecian Lecture, M. J. Lighthill [85] surveyed the field of sound generated aerodynamically. The author’s original theory of sound radiation fields which were by-products of airflows has been extended and improved by Oudle and Svensen Williams. It is explained in this lecture fully but simply, and used as a framework for short analyses of our experimental knowledge on pulse-jet noise, hydrodynamic sound generation, acoustic tones, propeller noise, and boundary-layer noise as well as for a somewhat extensive discussion of the noise of jets, both stationary and in flight. Improved knowledge of space-time correlations in turbulent flow is used to throw new light on the noise radiated by turbulent boundary layers, as well as by jets at the higher Mach numbers. Supersonic booms and the scattering of both sound and shock waves by turbulence are briefly touched upon. The lecture ends with a discussion of the methods used for the reduction of jet aircraft noise in the light of our knowledge of its physical basis.

Morgan, Sutherland, and Young [80] have studied the use of Acoustic Scale Models for Investigating Near Field Noise of Jet and Rocket Engines. Analytical and experimental studies were made to examine the feasibility of using acoustic scale models for near field noise investigations. Analyses show that the important characteristics of noise generation, propagation, and measurement can be scaled. A relatively few deviations from this involve small errors which are generally negligible in the near field. The most straightforward model is seen to be one which duplicates the gas flow parameters of the full scale engine. The validity of such models has been demonstrated by a series of exhausts.
and whether in a free field or in the presence of objects which interfere with the flow, such as shaped nozzles and flame deflectors. It is further determined both analytically and experimentally that models, in certain cases, may be simplified without impairing the results of a scaled test. Considerations in simplifying a model include: reduction of the nozzle size; absence or presence of reflecting surfaces; use of fewer than the full scale number of engines; and use of a substitute gas which is different from and at a lower temperature than that in the full scale engine.

In another experimental paper Bull and Willis [86] give some results of investigations of the surface pressure field due to a turbulent boundary layer. Experimental results for the space-time correlations of the fluctuating pressure field of a turbulent boundary layer are given, and an empirical representation of the pressure field suitable for structural response calculations is put forward. Variation with Mach number of r.m.s. pressure as a function of skin friction is given for Mach numbers up to about 1.6. The probability distribution of the pressure fluctuations at a fixed point in the field is found to approach closely to Gaussian. The acoustic power output from a boundary layer on a larger boundary surface is obtained as roughly $2.10^{12} \frac{L}{W} \frac{S}{\rho u^2}$. Spectra of boundary layer noise in two jet aircraft are presented and compared with the spectrum of the boundary layer excitation.

In an experimental paper Williams and Wedge [87] measured the fluctuating pressure at the wall beneath a thick turbulent boundary layer. The data include the mean-square pressure, power spectrum of the pressure, space-time correlation of the pressure transverse to the stream. The root-mean-square pressure at the wall was 2.10 times the wall shear stress. The power spectra of the pressure were found to scale with the free-stream speed and the boundary-layer displacement thickness. A few tests with a rough surface showed that the increase in root-mean-square wall pressure was greater than the increase in wall shear stress. The space-time correlation measurements parallel to the stream direction exhibit maxima at certain time delays corresponding to the convective pressure-producing eddies at speeds varying from 0.56 to 0.53 times the stream speed. The lower correlation speeds are measured when the spatial separation of the pressure transducers is small, or when only the pressure fluctuations at high frequencies are correlated. Higher correlation speeds are observed when the spatial separation of the pressure transducers is large, or when only low frequencies
are correlated. The result that low-frequency pressure fluctuations have the highest convective speed is in agreement with the measurements of Cerco [49,50] in a fully turbulent tube flow. Analysis of these measurements also shows that both large- and small-scale pressure-producing eddies decay after traveling a distance proportional to their scale. More precisely, a pressure-producing eddy of large or small wavelength $\lambda$ decays and vanishes after traveling a distance of approximately $\frac{\lambda}{2\lambda}$. The transverse spatial correlation of the wall-pressure fluctuations was measured and compared with the longitudinal scales of both large- and small-scale wall-pressure fluctuations. Both the transverse and the longitudinal scale of the pressure fluctuations were of the order of the boundary-layer thickness. The transverse and longitudinal scales of both large- and small-scale wall-pressure fluctuations were also measured and were found to be approximately the same.

In a similar report Wooldridge and Willmarth [88] measured the correlation between the fluctuating velocities and the fluctuating wall pressure in a thick turbulent boundary layer. They studied the noise from turbulence convected at high speed. The theory initiated by Lighthill for the purpose of estimating the sound radiated by a turbulent fluid flow is extended to deal with both the transonic and supersonic ranges of eddy convection speed. The sound is that which would be produced by a distribution of corrected acoustic quadrupoles whose instantaneous strength per unit volume is given by a turbulence stress tensor, $T_{ij}$. At low subsonic speeds the radiated intensity increases with the eighth power of velocity although quadrupole convection augments this basic dependence by a factor $\left|1 - M \cos \theta \right|^{-2}$, where $M$ is the eddy convection Mach number and $\theta$ the angular position of an observation point measured from the direction of eddy motion. At supersonic speeds the augmentation factor becomes singular whenever the eddy approaches the observation point at sonic velocity, $M \cos \theta = 1$. At that condition a quadrupole degenerates into its constituent simple sources, for each quadrupole element moves with the acoustic wave front it generates and cancelling contributions from opposing sources is essential in determining quadrupole behavior, cannot combine but are heard independently. This simple-source radiation is likened to a type of eddy Mach wave whose strength increases with the cube of a typical flow velocity. When quadrupoles approach the observer with supersonic speed sound is heard in reverse time, but is once again of a quadrupole nature and the general low speed result is again applicable. The limiting
high speed form of the convection augmentation factor is $|\mathbf{v} \cos \theta|^5$ which combines with the basic eighth power velocity law to yield the result that radiation intensity increases only as the cube of velocity at high supersonic speed. The mathematical theory is developed in some detail and supported by more physical arguments, and the paper is concluded by a section where some relevant experimental evidence is discussed.

Studies of rocket noise simulation with substitute gas jets and the effect of vehicle motion on jet noise was investigated by Morgan, Young [91]. The report is written in two parts. In part I the feasibility of using helium jets as a practical substitute for actual rockets in scale model acoustic tests was investigated by conducting an experimental program with four heated helium models. Sufficient evidence is presented to indicate that that substitute gas modeling concept is valid. In part II an investigation was made to determine the effect of flight vehicle motion on propulsion system noise which is propagated to parts of the vehicle located in the near field. Experimental results compare favorable with predictions based on the hypothesis which explains the effect of vehicle motion by two separate factors: (1) the noise produced by a jet in motion in dependent upon the relative velocity between the jet and the air through which it moves; and (2) a shifting of the noise radiation pattern toward the rear occurs because of the combined effects of vehicle motion and the finite velocity of sound.

The spectra and Directivity of Jet Noise is the topic of a short note by H. S. Hibber [96]. On the assumption of locally isotropic turbulence superposed on the mean jet flow, the broadly peaked noise spectrum is derived in terms of a sum of two bell-shaped spectra peaked on octaves apart. The proportions vary with direction $\theta$ from the jet axis being dominated by the bass spectrum at small angles and the treble spectrum beyond, say 70°. This is accomplished by a factor $\cos^2 \theta$ for the bass spectrum.

The question of possible sound produced by stresses at a rigid surface is discussed by Meacham [91,92,93]. It has been previously pointed out that these stresses won't produce sound. The error in the original analysis is shown to be a confusion between hydrodynamic pressure and sound field pressure.
Proves Williams [94] considers the noise produced by boundary layer turbulence in a short note. Some erroneous deductions made recently are discussed and a short survey of the relevant theories presented. It is pointed out that no fundamental errors exist in the basic theoretical work on this topic, and that some of the apparent inconsistencies are due to a misinterpretation of those theories.

The elements essential to a new vortex theory of edgestones are described by Alan Powell [95]. It is shown how both the feedback to the orifice and the sound radiation may be directly attributed to the velocity field of the vortex cast off by the edge together with the sympathetic circulation about it. An upper limit to the intensity of the edgestone is discussed when the edge is very small; it depends on the size of the edge. It is shown how the small edge used by Lamb and Richardson is quite adequate in size to support the flow pattern implied by the feedback explanations of the mechanism of edgestones. The theory is shown to be equivalent to the so-called acoustical one in which the action at the edge is represented by a dipole whose strength is associated with the fluctuating force sustained by the edge. While Powell states that the rift between these two approaches to the theory of edgestones may be considered closed, it is to be noted that each may continue to have its special attractions in respect to specified requirements.

The surface integrals of the solution to the homogeneous wave equation are discussed by Powell [96] with special reference to the use of the pressure independent of the sound speed and point multipole representation for small surfaces enclosing the source. It is shown how the multipole representation can be used for not small surfaces, so as to afford simplification in estimating such pressures, even though the series of multipoles may diverge. The corresponding use of such pressures in the homogeneous wave equation is examined. The general solution for surfaces of arbitrary size contains both the aforementioned surface integrals and a volume integral over the region interior to that closed surface. The former stands alone for small surfaces and closely represents real physical conditions. The volume integral may stand alone, provided that the net multipole strength is zero and that the region of integration extends far enough for the retained time-delay effects to cause the volume integral to converge. The volume integral has the mathematical interpretation of repairing the deficiency of the surface integrals when the fictitious pressure is used in them, and consequently is unconnected with the actual basic mechanism of the source. The mathematical...
discussion is supported and illustrated by direct use of monopole, dipole, and lateral quadrupole fields.

The question of possible sound produced by stresses at a rigid surface is again discussed by Powell [97]. It was previously pointed out that these stresses are effective sources, being the reaction to momentum fluctuations in a contiguous flow and with which the true sound sources are to be associated. There is no error in the original general formalism, and no confusion need exist between hydrodynamic pressure and sound-field pressure in the limits of very small surfaces (when net forces are to be directly associated with dipole radiation) and of very large plane surfaces (when the surface pressures merely represent reflections of the quadrupole flow noise).
Experimental Results

In the development of the field of aerodynamic noise a large quantity of valuable experimental work has been done. The difficulty of the area of turbulence has necessitated the appeal of workers in the field to empirical techniques and hence the experimental results have played a particularly significant part in the understanding of the phenomena. Some examples of these experimental results are provided to illustrate the general nature of the work which has been done.

In figure 1 Bews [58] demonstrated the contours of sound pressure level for a subsonic jet. Figure 2 is a plot of power-spectral density versus Strouhal number for supersonic air jets also from Bews [58]. In figure 3 the space-time correlations of the surface pressure field is on the surface of a wind tunnel are given. These are taken from a report by Dall and Willis [83]. Figure 4 shows the experimental results of Wiliams and Woolridge [37] giving longitudinal space-time correlations of the pressure at the wall in a fluctuating boundary layer. Figures 5-8 are taken from the same paper and show respectively the peaks in the longitudinal space-time correlation, the dimensionless power spectra of various height in the boundary layer, and various values of the space-time correlation of fluctuating velocity with fluctuating wall pressure. Figure 9 gives the spectrum of near-field boundary-layer noise for a rough cylinder due to Pukin, Korbachar and Keefe [52].


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(a) $v = \approx 5$; nozzle diameter $D$, 5 inches.

Figure 1. Comparison of predicted and experimental contours of sound-pressure level in one-third-octave bands for a subsonic jet.
Dimensionless power-spectral-density level, dB \( \left( \frac{\alpha \eta}{D \nu} \right)_0 = 1 \)

Figure 2: Dimensionless power-spectral-density functions for supersonic air jets.

Dimensionless power-spectral-density, \( \frac{\alpha \eta}{D \nu} \)
Figure 5 Space-time correlations of the surface pressure field in flow direction measured in wind tunnel.
Figure A. Peaks of the longitudinal space-time correlation in a low-end high-frequency band.
Figure 6. Dimensionless normalised power spectra of the longitudinal velocity components at various heights in the boundary layer. Spectra are normalised to have unit area in the frequency band of the subsequent experiments.
Figure 7. Measured values of the space-time correlation of fluctuating velocity with fluctuating wall pressure.
Figure 6. Measured values of the space-time correlation of fluctuating velocity with fluctuating wall pressure.
Figure 9. Spectrum of near-field boundary layer noise for rough cylinder with anti-correlated microphones 2 inches from surface. Data for 7000 rpm adjusted to 10,000 rpm on basis of $T^2 = T^3$. 