REVIEW OF PREVIOUS WORK ON SHORT-TIME TESTS FOR PREDICTING FATIGUE PROPERTIES OF MATERIALS

Franz H. Vitovec
and
Benjamin J. Lazan

University of Minnesota

August 1953

Materials Laboratory
Contract No. AF 33(038)-20840
RDO No. 614-16

Wright Air Development Center
Air Research and Development Command
United States Air Force
Wright-Patterson Air Force Base, Ohio

Approved for Public Release
FOREWORD

This report was prepared by the University of Minnesota, Minneapolis, Minnesota, under USAF Contract No. AF 33(038)-20840. The contract was initiated under Research and Development Order No. 614-16, "Fatigue Properties of Structural Materials", and was administered under the direction of the Materials Laboratory, Directorate of Research, Wright Air Development Center, with W. J. Trapp acting as project engineer.
ABSTRACT

Experimental observations relating to the fatigue process and theories of fatigue are briefly reviewed. Short-time fatigue testing methods are systematically reviewed and critically discussed. The classification of the methods is based on the relationship of fatigue properties to static properties, to stress-strain characteristics under reversed stress, and to other physical properties. Other methods discussed involve assumptions regarding the shape of the S-N curve. Also tests utilizing special loading conditions are reviewed. Advantages and applicability of the different methods are discussed.

PUBLICATION REVIEW

This report has been reviewed and is approved.
FOR THE COMMANDING GENERAL:

M. E. SORTE
Colonel, USAF
Chief, Materials Laboratory
Directorate of Research

WADC TR 53-122
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>II</td>
<td>Experimental Observations Relating to the Fatigue Process and Theories of Fatigue</td>
<td>2</td>
</tr>
<tr>
<td>III</td>
<td>Theories of Fatigue</td>
<td>4</td>
</tr>
<tr>
<td>IV</td>
<td>Relationship of the Fatigue Strength to Static Properties</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>4.1 Stress at Which Slip is Initiated</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>4.2 Static Proportional Limit</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>4.3 Static Yield Limit</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>4.4 Static Ultimate Tensile Strength (Apparent Value)</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>4.5 Static Tensile Strength (True Value)</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>4.6 Combination of Various Static Properties</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>4.7 General Evaluation of the Various Relationships Between Static and Fatigue Properties</td>
<td>11</td>
</tr>
<tr>
<td>V</td>
<td>Mathematical Representation of the S-N Curve</td>
<td>12</td>
</tr>
<tr>
<td>VI</td>
<td>Dynamic Proportional Limit</td>
<td>13</td>
</tr>
<tr>
<td>VII</td>
<td>Failure Stress Under Progressive Load Increase</td>
<td>14</td>
</tr>
<tr>
<td>VIII</td>
<td>Damping</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>8.1 The Mechanism of Damping in Metals at Low and High Stress</td>
<td>16</td>
</tr>
<tr>
<td></td>
<td>8.2 Relationship of Damping at High Stress to Fatigue Properties</td>
<td>19</td>
</tr>
<tr>
<td></td>
<td>8.3 Damping at Low Stress</td>
<td>22</td>
</tr>
<tr>
<td>IX</td>
<td>Change in Modulus of Elasticity</td>
<td>23</td>
</tr>
<tr>
<td>X</td>
<td>Magnetic Properties</td>
<td>24</td>
</tr>
<tr>
<td>XI</td>
<td>Electrical Resistance</td>
<td>24</td>
</tr>
<tr>
<td>XII</td>
<td>Surface Activity of Stressed Material</td>
<td>25</td>
</tr>
<tr>
<td>XIII</td>
<td>Effect of Prior Fatigue Stress on Ultimate Tensile Strength</td>
<td>26</td>
</tr>
<tr>
<td>XIV</td>
<td>Coefficient of Thermal Expansion</td>
<td>27</td>
</tr>
<tr>
<td>XV</td>
<td>Determination of the Fatigue Limit by X-Ray Diffraction</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>15.1 Method of F. Regler</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>15.2 Method of R. Glockner</td>
<td>28</td>
</tr>
<tr>
<td>XVI</td>
<td>Strength Under Combination of Alternating Bending and Tension Loads</td>
<td>28</td>
</tr>
<tr>
<td>XVII</td>
<td>Summary and Analysis of Methods</td>
<td>29</td>
</tr>
<tr>
<td>Bibliography</td>
<td>31</td>
<td></td>
</tr>
</tbody>
</table>

WADC TR 53-122

Approved for Public Release
<table>
<thead>
<tr>
<th>Figure</th>
<th>Illustration</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Fatigue Limit of Steel Versus Yield Strength According to Different Investigators, (A. Schaal)</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>Fatigue Limit Versus Tensile Strength of Several Steels, (E. Houdremont and R. Mailaender)</td>
<td>41</td>
</tr>
<tr>
<td>3</td>
<td>Fatigue Limit Versus Tensile Strength of Steel (Battelle Memorial)</td>
<td>41</td>
</tr>
<tr>
<td>4</td>
<td>Fatigue Limit Versus True Tensile Strength Based on Necked Area, (E. Herold)</td>
<td>42</td>
</tr>
<tr>
<td>5</td>
<td>Frequency Distribution of the Fluctuation Obtained from Different Formulas for the Determination of the Fatigue Limit from the Tensile Properties of Steel, (R. Mailaender)</td>
<td>42</td>
</tr>
<tr>
<td>6</td>
<td>Fluctuations Obtained from Different Formulas for the Determination of the Fatigue Limit from the Tensile Properties of Steel, (Lequis, Buchholz, Schulz)</td>
<td>42</td>
</tr>
<tr>
<td>7</td>
<td>Deviations of the Fatigue Limit Calculated from Different Formulas from the Origin (R. Mailaender)</td>
<td>43</td>
</tr>
<tr>
<td>8</td>
<td>Yield Strength Versus Ultimate Strength for Several Steels, (E. Houdremont and R. Mailaender)</td>
<td>43</td>
</tr>
<tr>
<td>9</td>
<td>Relationship Between Fatigue Limit and Tensile Strength of Steel with Different Ratios of Yield Strength to Tensile Strength, (A. Fry, A. Kessner, and R. Oettel)</td>
<td>43</td>
</tr>
<tr>
<td>10</td>
<td>Fatigue Limit Versus Ratio Yield Limit/Ultimate Strength of Several Steels, (M. Hempel and H. Krug)</td>
<td>44</td>
</tr>
<tr>
<td>11</td>
<td>Failure Stress Versus Destruction Velocity of Steel, (W. Spaeth)</td>
<td>45</td>
</tr>
<tr>
<td>12</td>
<td>Mean Stress Versus Mean Strain Under Constant Cyclic Stress Below the Fatigue Limit, (T. H. Smith)</td>
<td>45</td>
</tr>
<tr>
<td>13</td>
<td>Alternating Stress Versus Alternating Strain Under Progressively Increasing Load, (H. J. Gough)</td>
<td>46</td>
</tr>
<tr>
<td>14</td>
<td>Static and Dynamic Proportional Limit Versus Number of Cycles of Mild Steel, (B. J. Lazan and T. Wu)</td>
<td>46</td>
</tr>
<tr>
<td>15</td>
<td>Failure Stress Versus Square Root of Loading Rate of SAE 4340 Steel (E. J. Ward and D. C. Schwartz)</td>
<td>47</td>
</tr>
<tr>
<td>16</td>
<td>Failure Stress Versus Square Root of Loading Rate of Various Materials. Stepwise Load Increase (Sinclair)</td>
<td>48</td>
</tr>
<tr>
<td>17</td>
<td>Specimen Temperature Versus Alternating Stress for Different Kinds of Steel, (Putnam and Harsch)</td>
<td>49</td>
</tr>
<tr>
<td>18</td>
<td>Damping Energy Versus Magnitude of Alternating Stress, (E. Lehr)</td>
<td>49</td>
</tr>
<tr>
<td>19</td>
<td>Temperature Rise of the Specimen Versus Number of Cycles at Different Constant Stresses. Cr-Ni-Steel. (P. Ludwik)</td>
<td>50</td>
</tr>
<tr>
<td>20</td>
<td>Temperature Rise of the Specimen Versus Number of Cycles at Different Constant Stresses. Carbon Steel with 68,300 psi. Ultimate Strength (P. Ludwik)</td>
<td>50</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>21</td>
<td>Damping Energy Versus Number of Cycles at Constant Shear Stress. Fatigue Strength 19,200 psi. (P. Ludwik and R. Scheu)</td>
<td>51</td>
</tr>
<tr>
<td>22</td>
<td>Temperature Rise of the Specimen Versus Number of Cycles at the Fatigue Limit of Steels with Different Carbon Content, (P. Ludwik)</td>
<td>51</td>
</tr>
<tr>
<td>23</td>
<td>Temperature Rise of the Specimen and Damping Energy Versus Number of Cycles for Various Steels. Influence of Cooling of the Specimen on the Damping Energy (P. Ludwik and R. Scheu)</td>
<td>52</td>
</tr>
<tr>
<td>24</td>
<td>Frequency of Definite Deviation Between Strength Based on Damping Tests and Wohler Fatigue Strength as a Function of the Ultimate Strength of Steel (W. Herold)</td>
<td>53</td>
</tr>
<tr>
<td>25</td>
<td>Logarithmic Decrement and Natural Frequency Versus Alternating Stress, (E. Gerold and A. Karius)</td>
<td>53</td>
</tr>
<tr>
<td>26</td>
<td>Effect of Reversed Stress Magnitude and Stress History on Damping Energy of Stress-Relieved Steel, (B. J. Lazan and T. Wu)</td>
<td>54</td>
</tr>
<tr>
<td>27</td>
<td>Effect of Stress History on Static and Dynamic Stress-Strain Curves for Mild Steel (B. J. Lazan and T. Wu).</td>
<td>54</td>
</tr>
<tr>
<td>28</td>
<td>Electrical Resistance and Temperature of the Specimen Versus Alternating Stress for Monel-Metal and Copper, (F. H. Moore and S. Konzo)</td>
<td>55</td>
</tr>
<tr>
<td>29</td>
<td>Rate of Electron Radiation Versus Alternating Stress, (J. Kramer)</td>
<td>56</td>
</tr>
<tr>
<td>30</td>
<td>Effect of 1,400,000 Cycles of Reversed Flexural Stress on Ultimate Tensile Strength (H. F. Moore and H. B. Wishart)</td>
<td>56</td>
</tr>
<tr>
<td>31</td>
<td>Coefficient of Thermal Expansion Versus Alternating Stress, (J. L. Rosenhoitz and D. T. Smith)</td>
<td>57</td>
</tr>
<tr>
<td>32</td>
<td>Surface Stress of Steel Determined by X-Rays Versus Number of Cycles at Different Alternating Stresses.</td>
<td>57</td>
</tr>
<tr>
<td>33</td>
<td>S-N Curve Under Constant Bending Strain and Variable Static Tensile Stress (E. Mohr)</td>
<td>58</td>
</tr>
</tbody>
</table>
Contrails
The determination of the fatigue properties of materials by the usual Wohler Method is needless to say a time consuming job. Not only is it important that a sufficient number of specimens be used so that the resultant S-N curve is statistically sound, but also is it necessary to carry some specimens as far as $10^8$ cycles for some materials.

One obvious way of accelerating the usual S-N fatigue test is to increase the testing frequency. Although this increase in testing frequency does not usually affect the fatigue properties more than a few percent there is usually a definite upper limit imposed by the temperature increase caused by internal damping. Furthermore, unknown inertia forces and other sources of error in fatigue testing sometimes become increasingly serious with higher frequency. However, even if testing frequency is increased to the highest practical limit, the performance of the usual S-N fatigue test is still a time consuming task, particularly if statistical soundness of the results is important.

In view of the importance of reducing fatigue testing time there has been considerable interest for several years in accelerated fatigue testing methods; that is methods for rapidly determining the fatigue properties of materials without completing the usual S-N fatigue diagram. Some methods for rapidly determining the fatigue properties of materials are based on the relationship between the fatigue limit and other physical properties of the material. Other methods are based on the behavior of a material under various loading histories. The validity of such relationships cannot be definitely established until more is known about the mechanism of fatigue and the relationship of fatigue properties to other physical properties. Thus, until a more basic understanding of fatigue and other mechanical behavior is developed, all short time fatigue tests must be based primarily on empirical relationships.

Before reviewing the various types of accelerated fatigue tests it is desirable first to discuss briefly the mechanism of fatigue. This is done in the next section.
SECTION II. EXPERIMENTAL OBSERVATIONS RELATING TO
THE FATIGUE PROCESS AND THEORIES OF FATIGUE

The results of metallographic and X-ray investigation on the
mechanism of fatigue may be summarized as follows.\(^1\)

a. The plastic deformation under repeated stress occurs by slip
and twinning along the same crystallographic planes and direc-
tion as under static stresses.

b. Slip follows the same basic law under both static and alter-
minating stress (the law of maximum resolved shear stress)
and is accompanied by strain hardening (1).\(^2\)

c. The reversed stress also reverses the direction of the defor-
mation. However, slip in the reversed direction does not seem
to take place along the identical slip plane, but along a neigh-
boring one. Hence, slip in opposite directions leaves the
external shape of the crystals essentially unchanged (2).

d. Fatigue cracks always start in regions which have suffered
the most severe deformation, that is, along the operative,
twin, or cleavage plane. The cracks then spread along the
path of least resistance, that is, along planes of actual maxi-

mum shear stress or along slip planes.

e. The nature of fatigue failure in polycrystalline aggregates is
the same in fatigue as under static stresses, namely inhibition
of slip (3). The cracks start in a highly deformed portion of
a crystal from which they spread. Cracks rarely seem to
follow a grain boundary and the rate of propagation of a crack
is generally reduced in the neighborhood of a boundary.

\(^1\) This outline follows in the main points the publication of W. Boas,
"Theories of the Mechanism of Fatigue Failure, The Failure of Metals by

\(^2\) Numbers in parenthesis refer to the Bibliography appended to this
paper.
f. In polycrystalline materials, fatigue cracking seems to start early in the fatigue life, as low as 5% of the cycles to failure. The crack which leads to fracture is generally not the first crack which appears. The crack propagates slowly at first, and then after a certain number of cycles progresses at an increasingly rapid rate until fracture occurs (4).

g. After deformation under static stresses, the lattice distortions are found rather uniformly throughout the volume of the specimen, whereas under fatigue conditions the lattice distortions are localized in regions where fatigue cracks form (5).

h. In the surface of multi-crystalline materials, residual compressive stresses are developed by any kind of alternating stresses (6).

i. Little is known about the cause of the initiation of the fatigue crack. There have been several attempts to relate the start of the fatigue crack to the mechanism of the deformation. But all of the existing theories include points which either cannot be accepted or are vaguely expressed. Thus, none of the existing theories seems to be fully satisfactory.

The deformation of a polycrystalline material is in any case inhomogeneous, due not only to the statistical orientation of the grains, but also to the influence of the free surface. In polycrystalline materials, the deformation of each crystallite is hindered by the influence of its neighboring crystallites. This hindering of flow results from the mutual effect of the grains. The greater the deviation in crystallographic orientation between two grains, the greater is their mutual resistance to flow (7). Grain size and spacing are also important factors determining the mutual resistance to flow offered by adjacent crystals.

The total hindering effect of the environment surrounding any crystal upon its deformation can thus be estimated by the summation of the effects of a finite number of successive surrounding spherical layers (8). For crystallites near a free surface, a part of this spherical field of influence is missing, and crystallites actually on the surface are affected only by a hemispherical field. Since these grains are less hindered to deform because they have fewer neighbors, they respond plastically under stress which is locally smaller than that required to deform the grains in the interior of the body. Plastic deformation will thus occur first on the surface and be propagated from the surface toward the inside of the material (9).
The observations regarding the precendency of surface deformation are based on a large number of X-ray examinations. It has been found in static tension, compression, and bending tests made on steel and aluminum that the layers nearest to the free surface of the specimens start to yield under stresses which are on the average one-half to two-thirds as great as the yield stress for uniform yielding (10). Nevertheless, on account of the statistical fluctuation, yielding occurs in some crystals of the surface layer at stresses in the range of the yield strength of single crystals (11).

These facts are of great importance in the interpretation of fatigue behavior. They explain why the fatigue cracks always start from the surface of the specimen, even under axial loading having uniform stress distribution. Also this observation helps to explain the relationship between the static yield limit and the fatigue limit. Finally, the high influence of surface conditions on the fatigue limit becomes apparent.

Further evidence that fatigue takes place in a thin layer near the surface is given by the experiments of E. Siebel and G. Stahli (12). The life of a fatigue specimen could be greatly increased by removing a thin layer (0.002") of the specimen by polishing or etching after a certain number of cycles.

SECTION III. THEORIES OF FATIGUE

As discussed previously practically all short-time fatigue tests are based on the relationship between fatigue properties and the other physical and mechanical properties of the material. A complete understanding and utilization of this relationship is possible only if it is considered in the light of theories regarding the mechanism of fatigue. It is desirable therefore to discuss briefly the three main groups of theories on the mechanism of fatigue.

The first group of theories is based on the observation that the imperfections and elastic anisotropy of the crystallites in polycrystals cause stress concentration. It is assumed that work hardening resulting from deformation causes an increase of the stress concentration until the stress in the deformed region exceeds the static value required for rupture, thus providing initiation for the fatigue crack (13) (14).
The second group of theories assumes a successive disruption of interatomic bonds by alternating stresses. The number of bonds disrupted increases rapidly with increasing number of cycles (15).

In the third group an accumulation of lattice distortions caused by alternating slipping is assumed. The lattice distortions result in a development of internal stresses. After a definite alternating deformation the severity of these internal stresses, when superimposed upon the nominal stress, is sufficient to crack the material (16-20).

From a basic viewpoint these three groups of theories are not as different as a casual inspection might indicate. It does not appear likely that one of these theories alone can explain the mechanism of fatigue; all three theories considered together offer more opportunity. There is no doubt that alternating slipping results in lattice distortion and internal stresses on a submicroscopic scale and the nominal stress is superimposed over these internal stresses. Also imperfections and the anisotropy of the crystallites results in a stress concentration which is superimposed on the effect of increasing lattice distortion. After a certain number of alternating stress cycles the material is so weakened by internal stresses that a microscopic crack is initiated.

In recent years there has been active development of the concept that the mechanism of slip takes place by propagation of a special kind of lattice distortions, the so-called "dislocation". The rate of deformation is then determined by the number of dislocations propagated. It is assumed that shear hardening is caused by arresting an increasing number of dislocations in the slip plane. This increasing density of dislocations should also take place in reversed shearing. The concentration of similar dislocations in the slip plane results in a complicated stress field. Shear forces occur between the two blocks on each side of the slip plane and cause a bending of the planes and tension stresses which are perpendicular to the slip plane.

The dislocation itself is surrounded by a stress field. In this region Hooke's law is no longer valid and must be replaced by a nonlinear and unsymmetrical relationship. Tensile forces produce a greater displacement than compressive forces of the same magnitude. Therefore, dislocations cause an increase in the volume in the slip zone and a decrease of the cohesive strength of this zone. Although increase of the volume is small (21), since the volume of the slip zone is relatively small compared with the total
volume of the specimen, it results in compressive internal stresses in the slip region. Because of the free surface influence discussed previously, slip takes place originally only in a thin surface layer at fatigue stresses. This explains that internal stresses arise during alternating stressing with a compressive stress in this surface layer. These compressive stresses are superimposed on the stress field surrounding the slip plane. Below the fatigue limit it appears that the compressive stresses grow faster than the additional tensile stresses perpendicular to the slip plane and thus a crack cannot be initiated. Beyond the fatigue limit, the cohesion is so weakened by the increasing number of dislocations that cracking is initiated under the external loading before the balance between the increasing compressive stresses and the internal tension stresses is reached. Whether or not a crack propagates under these conditions depends on the initial depth of the crack and the extent of the region in which the cohesive strength of the material is weakened by reversed shearing.

The crack itself causes a stress concentration. The amount of the concentration depends upon the depth of the crack and the radius of the root of the crack. It is noted that in real material the radius of the root of a crack or notch never could be smaller than the atomic distance. Therefore, the peak stress never could be infinite at finite average stress, as it is claimed by the classic theory of elasticity for a continuous medium (22). Finally, for notches with such a small radius of the root, the cohesive strength of the material associated with the attractive forces of the atoms must be taken into consideration rather than the actual tensile strength of the material. This cohesive strength is on the average 1000 times greater than the tensile strength. It is only decreased by the internal residual stresses in the small region of dislocations. Whether a crack does or does not propagate depends therefore upon the initial size of the crack, the natural cohesion of the perfect crystal, the lattice condition in this region, and the nature and amount of loading. This may explain why fatigue cracks sometimes are initiated under alternating stresses but do not propagate.

The mechanism of fatigue described above is, needless to say, very complicated. However, the understanding provided by this background provides an operating framework for better understanding the relationship between static and fatigue properties.
In principle, the fatigue limit is marked by the alternating stress under which the reversed plastic straining practically ceases due to the influence of strain hardening. This should result in changes in the physical properties of the material such as damping, electrical resistance, magnetic properties, coefficient of expansion. Some of these changes may, of course, be so small as to be unmeasurable using presently available techniques. In considering these changes it is necessary to consider not only stress magnitude, as is done in most accelerated fatigue tests, but also stress history.

Summarizing the above discussion insofar as it relates to accelerated fatigue testing, the behavior of a material under fatigue stress is a function of:

a) The constitution, such as crystal system, composition, state of solution, grain size.

b) Deformation characteristics, such as yield limit of single crystals and grain boundaries and strain hardening.

c) The surface finish, such as strain hardening by cold working, or age hardening, etc.

d) The shape of the surface and of the specimen. These facts influence not only the local yield point and the local stress distribution, but also govern the macroscopic stress distribution caused by different loading of the specimen.

It is thus apparent that understanding and predicting the reliability of a short-time fatigue test is a very complicated problem. Practically every property of a material is probably related in some way to its fatigue strength, but this relationship generally differs from material to material. Thus, the problem of short-time fatigue testing appears to be that of finding an easily and quickly measured physical property which possesses a constant relationship to the fatigue limit of the material. Various possibilities are suggested from the discussion of the mechanism of fatigue, and experimental evidence regarding the reliability of such associations are presented in the next section.
SECTION IV. RELATIONSHIP OF THE FATIGUE STRENGTH TO STATIC PROPERTIES

4.1 Stress at Which Slip is Initiated

W. Rosenhain has suggested (23) that one method of determining the fatigue limiting range would be to determine the stress required to produce slip bands in the material. The execution of such a test would involve the examination of polished specimen during fatigue test at various stress magnitudes. It is now known from X-ray, metallographic, and other evidence that the first signs of permanent deformation do not mark the fatigue range.

4.2 Static Proportional Limit

Numerous experiments published by H. F. Moore and J. B. Kommers (24), R. R. Moore (25), H. F. Moore and T. M. Jasper (28) and J. M. Lessels (27) showed that the fatigue limit of all kinds of metals lies beyond the static proportional limit and no constant relation between these two properties seems to exist. The observation of this relationship is as expected since localized strain hardening caused by slip is likely to have a pronounced effect on fatigue which could not be revealed by the static test. Furthermore, the location of the apparent proportional limit depends greatly upon the accuracy of testing equipment; in fact, the true proportional limit is probably at zero stress (since damping, an indication of non-linearity between stress and strain, is displayed at even very low stresses).

4.3 Static Yield Strength

A. Schaal (28) (29) investigated the relation between the fatigue strength and the yield strength of steel, and an aluminum alloy determined by both static tests and by X-ray methods. He found, in agreement with other investigators, that the ratio of fatigue to yield strength decreases with increasing yield strength as shown in Figure 1. In non-ferrous metals, having no definite yield point such as steel, the 0.2% off-set yield strength was arbitrarily assumed to be equivalent to the yield strength. P. Ludwik (30) also investigated the ratio of this yield strength to the fatigue limit and found this ratio to be 0.33 for aluminum alloys, more than 0.5 for Cr-Ni steel, 0.75 for mild steel, 1.4 for brass, and more than 2.0 for pure copper.
4.4 Static Tensile Strength (Apparent Value)

The first attempts to find a relationship between the apparent tensile strength of a material and its fatigue limit were made by L. Spangenberg (31), C. E. Stromeyer (32), and H. F. Moore and F. B. Seely (33). R. Mailaender (34) also investigated this ratio (Figure 2) and after numerous experiments arrived at the following relationship between the tensile strength $S_t$ and the fatigue limit $S_f$ of steel:

$$S_f = 0.47 \ S_t \pm 20\%.$$

A. Pomp and M. Hempel (35) investigated the ratio of fatigue limit to tensile strength of steel by statistical methods and found an average value of 0.463 between the limits of 0.44 and 0.52. In general, the conversion factor seems to decrease with increasing tensile strength as shown in Figure 3.

4.5 Static Tensile Strength (True Value)

W. Herold (36) studied the relationship between the fatigue limit $S_f$ and what is usually called the true tensile strength $S_t'$ determined by dividing the breaking load by the reduced area at the section of impending fracture. He proposed the relationship

$$S_f = 0.25 \ S_t'.$$

The scatter in the data he procured in relationship to the expression is shown in Figure 4. It appears there that this relationship is not more reliable than that based on the apparent tensile strength. Since the true tensile strength is more difficult to obtain accurately than the apparent tensile strength, this approach does not appear to offer much promise.
4.6 Combination of Various Static Properties

Several formulas developed to permit calculations of the fatigue strength $S_f$ of steel from the yield point $S_y$ and the other tensile properties, are listed below.

a) $S_{fb} = 0.65 S_y \pm 30\%$  \hspace{1cm} (37)
b) $S_{fb} = 0.285 (S_y + S_t)$  \hspace{1cm} (38)
c) $S_{fb} = 0.25 (S_y + S_t) + 5$  \hspace{1cm} (39)
d) $S_{fb} = 0.20 (S_y + S_t + \psi )$  \hspace{1cm} (40)
e) $S_{fb} = 0.175 (S_y + S_t + \delta_{10} + 100)$  \hspace{1cm} (41)
f) $S_{fb} = \alpha S_t + \beta S_y$,  \hspace{1cm} (42)

$$\alpha = \frac{1}{185} (0.5 S_t + 7), \quad \beta = \frac{1}{183} (142 - S_t)$$

where $S_y$ = yield stress, $S_t$ = tensile strength, $\psi$ = reduction in area, $\delta_{10}$ = strain to fracture, and $S_{fb}$ = fatigue limit under cyclic bending.

The reliability of these formulas has been evaluated by hundreds of tests on several kinds of steel including plain carbon steel, cast steel, and various alloy steels containing Mn, Cr, Ni, W and other elements. Figures 5, 6, and 7 show the results of some of these tests and a statistical evaluation (39), (41), (42), (28), (Figure 7). A. Fry (42) also studied the reliability of these equations representing the following results in the deviation of the different formulas from the experimental results:

<table>
<thead>
<tr>
<th>Formula</th>
<th>b</th>
<th>d</th>
<th>e</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deviation %</td>
<td>13.0</td>
<td>6.7</td>
<td>5.6</td>
<td>4.6</td>
</tr>
</tbody>
</table>

M. Hempel and H. Krug (44) investigated the relation between the fatigue strength and the ratio of yield strength to ultimate strength of steel. It was found that this relation depends on the type of treatment which leads to an increase of the ratio; a heat treatment which increases the yield point also generally increases the fatigue strength, but a cold-working treatment does not appear to influence the fatigue limit (45). For materials with an ultimate strength $S_t$ of 140,000 psi treatments which increase the yield
strength do not appear to effect significantly the fatigue strength. Some of the data on this relationship are given in Figures 8, 9, and 10.

4.7 General Evaluation of the Various Relationships by Static and Fatigue Properties

Since the ratio of the yield strength to the ultimate strength of a material differs over a wide range, the fatigue limit cannot, of course, bear a constant relationship to both static properties. Experimental results to date indicate that neither of these two properties has a definite relation to the fatigue limit for all materials. As discussed previously, the fatigue limit is primarily determined by the strain limit of a thin surface layer of the specimen. It is possible of course, that there are some relationships between this strain limit and the yield or ultimate strength of a material. However, the change of the properties of the thin layer would have only a small influence on the average properties revealed by the tension test. It is not surprising that no general relationship exists between the static tensile properties and fatigue.

There does appear to be an approximate relationship between static tensile and fatigue properties within definite classes of materials, for example, steel with some definite treatment. In justification of this it may be said that X-ray measurements have shown a relationship between the average yield strength of the thin surface layer and the yield of an entire steel specimen. Therefore, it is possible that a relationship also exists between the fatigue strength and the yield strength, provided that steels with similar heat treatment are investigated.

This relationship loses its validity for materials without a definite yield point. A yield strength arbitrarily established at 0.2% offset seems to be affected more pronouncedly by the type of treatment, such as cold working or heat treatment, than the fatigue properties.

Summarizing, there appears to be an approximate relationship between the fatigue strength of steel and its static tensile properties. However, to date such a relationship has not been found for other materials.
SECTION V. MATHEMATICAL REPRESENTATION OF THE S-N CURVE

C. E. Stromeyer (46) found that the S-N curve of steels could be represented by the formula:

\[ S = S_f + C \left( \frac{10^6}{N} \right)^{1/4} \]

where \( S \) is the fatigue limit, \( S_f \) is the definite fatigue limit, \( N \) is the number of cycles, and \( C \) is a constant for the material. From the results of two tested specimens the fatigue limit can be calculated from the equation:

\[ S_f = \frac{S_2 \left( \frac{10^6}{N_1} \right)^{1/4} - S_1 \left( \frac{10^6}{N_2} \right)^{1/4}}{\left( \frac{10^6}{N_1} \right)^{1/4} - \left( \frac{10^6}{N_2} \right)^{1/4}} \]

This formula seems to be reliable for steels if the two specimens break at approximately 50,000 cycles, and 300,000 cycles or more. The equation is inaccurate if both specimens break in less than 40,000 cycles. This formula suggests the existence of a very definite fatigue range and is therefore limited to steel.

A similar relationship was assumed by W. Spaeth (47), whose assumption, although not expressed mathematically, is indicated by the following equation:

\[ S = S_f + C \frac{N_R}{N} \]

where \( S \) is the failure stress, \( S_f \) the fatigue strength, \( N_R \) an arbitrarily chosen number of cycles, \( N \) the number of cycles to fracture, and \( C \) a constant. The ratio \( N_R/N \) is denoted by W. Spaeth as "destruction velocity." A linear relationship between \( S \) and \( N_R/N \) within the limits \( 1 = N_R/N = 0 \) was obtained for steel K 20 with the reference number of cycles \( N_R = 10^6 \) (see Figure 11) and for several lead alloys with \( N_R = 10^7 \). Several aluminum alloys give, with \( N_R = 10^7 \), bent curves which seem to meet at 6000 psi in the
case of $N_R/N = 0$. This indicates that aluminum alloys show a definite fatigue limit which differs from zero in the same way as steel.

Spaeth assumes basically that the S-N curves are hyperbolic in form and his procedure provides a method for extrapolation. This method has doubtful value as an accelerated fatigue test since a large number of cycles to failure are required for a reliable extrapolation. Other formulas for an analytical representation of the S-N curve have been derived by several authors. The relationship most often used at this time was proposed by W. Weibull (48):

$$N = k(S - S_f)^m,$$

where $S$ is the failure stress, $S_f$ the fatigue limit and $N$ the number of cycles to failure; $k$ and $m$ are constants. This formula may be valid only for steel, since it implies a definite fatigue limit of the material.

SECTION VI. DYNAMIC PROPORTIONAL LIMIT

J. H. Smith (49) attempted to associate the fatigue limit of a material with its dynamic proportional limit obtained by a special procedure. These procedures involved loading the specimen with a constant alternating stress smaller than the fatigue strength and then superimposing static pre-loading of increasing magnitude. If under these conditions the strain at the mean stress is plotted against the static mean stress, the relationship shown in Figure 12 results. At a definite amount of the preload the curve deviates from a straight line and Smith concluded that this dynamic proportional limit defines the fatigue range. Since Smith's results are not conclusive for a variety of materials, and since his method has received little of subsequent attention, this method must at present be given a doubtful classification.

H. J. Gough (50) determined a dynamic proportional limit under reversed cyclic stress by measuring alternating strain during progressively increasing alternating stress as shown in Figure 13. He found a linear stress-strain relationship below a certain stress beyond which the strains increase more rapidly than the stresses. He suggested that this dynamic proportional limit is a good indication of the reversed fatigue strength.
E. Kaufman (51) designated this limit obtained by Gough as the "alternating yield limit". Similar experiments have been made by McAdam, Jr. (52). T. Robson (53) tested specimens by this method in a kind of rotating cantilever beam. J. M. Lessels (54) found that at stresses beyond the fatigue limit a horizontal deflection of the specimen takes place, caused by damping. W. Mason's (55) experiments indicate that the fatigue limit is greater than the stress corresponding to the limit of the straight line portion of the load-slope diagram. B. J. Lazan and T. Wu (56) found that for a virgin mild steel specimen the dynamic proportional limit fell within the fatigue scatter band. However, the dynamic proportional limit was found to be decreased considerably by cyclic stress history above 84 per cent of the fatigue limit as shown in Figure 14. The figure also shows that cyclic stress below the fatigue limit does not affect the initial tangent modulus of the mild steel whereas the cyclic stress above the fatigue limit progressively decreases the initial tangent modulus.

In general, the experiments show that the dynamic proportional limit, although usually somewhat higher than the fatigue limit of steels and some non-ferrous metals, does provide a reasonably good approximation for many materials. However, some materials, such as aluminum alloys, do not reveal a dynamic proportional limit even at stresses well above the fatigue limit.

In considering the relationship between the dynamic proportional limit and the fatigue limit the several factors which affect the proportional limit must be carefully considered. For example, strain-hardening effects caused by stress history and the effect of strain rate may be very pronounced. Furthermore, as in all proportional limit determinations, the accuracy of testing affects significantly the points where nonlinearity is first observed.

SECTION VII. FAILURE STRESS UNDER PROGRESSIVE LOAD INCREASE

Recently M. Prot (57) developed an accelerated fatigue testing method which imposes a relatively small number of cycles on many specimens, thus enabling a better statistical analysis of variability. The specimens are subjected to a continuously increasing alternating stress until
fracture occurs. The rate of stress increase is constant during a given test, but may be different for different tests. Prot's method involves plotting the mean values of the failure stress at each rate of load against the square root of the loading rate. He proposes drawing a straight line through these points, extrapolating this line to intersect the ordinate of zero loading rate, and suggests that this indicates the fatigue limit. Prot (59) claims that this method would give more precise results than the regular Wohler method for any kind of material, metallic or non-metallic.

Prot's method was used for comparative tests of SAE 4340 blade steel by F. B. Stulen and W. Lamson (60). Prot's method has also been used by Ward and Schwartz (58), some of the results of this work being indicated in Figure 15. The tests proved that the endurance limit of that steel can be predicted with "reasonable accuracy" by the progressive-load method for this material. It appears that the fatigue limit as determined by this short-time test may be about 6 to 8 per cent lower than the value found by the regular Wohler test. This is of the usual range of deviations obtained by the short-time test methods in general.

E. M. Prot based his method theoretically on the assumption that the S-N curve is a hyperbola which is asymptotic to the fatigue limit. He further assumes that "the number of ruptures, molecules per cycle" is proportional to the difference between the test stress and the fatigue limit. This difference increases with the number of cycles in the Prot test method. Prot derived relationships which indicate that if the failure stress is plotted against the square root of the loading rate, a straight line should result. The formula has the form:

\[ S = S_f + k \alpha^{1/2}, \]

where \( S \) is the failure stress corresponding to the particular loading rate, \( S_f \) is the fatigue limit, and \( k \) is a material constant.

D. L. Henry (61) investigated the Prot method mathematically based on the theory of cumulative damage under repeated loads, developed by M. A. Miner (62) and the statistical theory of W. Weibull (63). By his calculations he obtains the formula:

\[ S = S_f + D \cdot \alpha^{-\frac{1}{m-1}}, \]
where $S$ is the failure stress corresponding to the particular loading rate $\alpha$, $E$ is the fatigue limit, $m$ and $D$ are characteristic constants of the material. According to Prot's theory $m$ should be 1. However, $m$ is generally found to deviate from 1. For some materials two $m$ values are needed to describe the characteristic curve. Recent data indicate the results of the Prot test may be influenced considerably by prestressing and coaxing effect. G. M. Sinclair (64) made similar experiments in which load was increased stepwise to investigate the coaxing effect in fatigue. Although Sinclair did not include Prot method consideration in his work, the data are replotted in Figure 16, to show failure stress versus average loading rate during the step-load increase. This figure indicates an agreement with Prot's assumption for 75S-T6 aluminum alloy and 70-30 brass which are not susceptible to strain aging according to Sinclair. However, materials such as SAE 2340 and 1045 steels, which are susceptible to strain aging display such a large coaxing effect as to make the Prot method misleading.

SECTION VIII. DAMPING

8.1 The Mechanism of Damping in Metals at Low and High Stress

The general characteristic of damping is the increase of the entropy. This increase can be associated with magnetic and mechanical effects. Mechanical damping is caused by the relief of initially produced stress differences by transportation. The two types of mechanical damping are caused by:

a) transportation of matter (diffusion, imperfections, slipping),
b) transmission of energy (thermal energy).

In discussing the mechanism of damping it is necessary to distinguish between damping at low stress, (say $>10$ psi) which may be assumed to occur without slipping, and damping at high stresses (say $>10^3$ psi) where deformation by slipping becomes significant.

The causes for damping at low stress (65) are discussed briefly below:

a. Microscopic thermo-elastic damping. Adiabatic compression
of an elastic material increases its temperature, adiabatic tension decreases it. Due to the elastic anisotropy of the grains, fluctuating stress gradients occur in polycrystalline materials which result in the transmission of thermal energy during alternating stress. This leads to a microscopic thermo-elastic damping. Stress gradients are also caused by submicroscopic and microscopic discontinuities in the material which lead to the same kind of damping.

b. **Macroscopic thermo-elastic damping.** This has the same physical interpretation as the microscopic thermo-elastic damping, but the stress gradients are on a large scale and caused by non-uniform stress distribution which depends on both the type of loading and specimen shape (for example, alternating bending or tension on notched specimens).

c. **Damping by diffusion.** Not only do stress gradients cause thermo-elastic damping, but they also influence the direction of migration of the atoms. If, for example, a cell of the lattice is enlarged by tensile stresses, the diffusion takes place easier in the tensile direction than in the contracted direction (66). Alternating stresses cause fluctuating migration of atoms which consumes mechanical energy and causes damping. The diffusion concept covers not only the migration of atoms, but also that of imperfections such as dislocations.

d. **Magnetic damping of ferromagnetic materials.** When a ferromagnetic body is stressed the magnetic domains having preferred orientation to the directions of stress are irreversibly enlarged. At higher stresses, the domains reorient themselves in the direction of stress. This is accompanied by the generation of eddy currents on a microscopic scale which involves a dissipation of energy and thus becomes a source of damping. This component of internal friction may be greatly decreased by placing the material in a strong magnetic field which holds the domains firmly in the direction of stress thereby minimizing the domain motions which cause eddy current. Cold working also tends to stabilize the domain orientations by internal stresses.

Although all the above phenomena contribute to damping at high stress, other causes dominate. As stresses are increased plastic deformation is initiated on an increasing scale even at stresses far below that limit normally determined as the yield strength. Several important characteristics of this phenomena are discussed below.
a) More than 90 per cent of the total energy input during static plastic deformation is transferred to heat while the rest is stored in the material by increasing lattice distortions. There is some evidence that the same is true under cyclic stress of large magnitude.

b) The generated heat increases the thermo-elastic damping.

c) The generated heat increases the temperature of the specimen. The amount of temperature rise is a function of the type of specimen (material, shape), type of testing machine (heat flow etc.), type of loading (stress distribution) and frequency.

d) Since in some cases the increase in specimen temperature is significant, a higher rate of diffusion occurs, resulting in a higher diffusion damping and recovery from lattice distortion. This recovery will tend to decrease damping. Furthermore, the increase of temperature decreases the yield strength which results in a larger plastic deformation, a higher generation of heat, etc.¹.

e) Frequency of cyclic stress influences all types of damping. At stresses and frequencies normally used in fatigue tests the effect of strain rate on plastic deformation is the most important factor. Increasing strain rate increases the yield strength rapidly at first, and then more slowly after a certain critical range has been reached (68, 69). Thus, the damping energy is generally larger at lower frequencies than at higher frequencies. Generally fatigue tests are conducted in the hypercritical range where the influence of frequency on the yield strength is relatively small.

At stresses near the fatigue limit the amount of unit damping caused by plastic deformation is much higher than that due to other weaker causes of damping.

¹ Cooling of the specimen decreases the damping energy (67).
8.2 Relationship of Damping at High Stress to Fatigue Properties

Several decades ago experiments were performed to determine if the fatigue limit of a specimen was indicated by a marked change in the damping properties of a specimen. The damping was measured by different methods:

a) by decay of free oscillations, providing the logarithmic decrement,

b) by measuring the energy of damping through a mechanical or electrical system,

c) by measuring the generated heat (only a part of the total energy imposed to the specimen).

One of the first who investigated the relation between damping and fatigue was O. Bouduard (70). With his machine he observed that marked difference in damping occurred only when the specimen was on the point of fracture. Further experiments on this problem were performed by F. E. Rowett (71) and B. Hopkinson and G. Trevor (72). W. E. Dalby (73) using a vibration decay method determined the fatigue limit as the lowest stress which ultimately produces a hysteresis loop. This conclusion was not confirmed by the experiments of H. J. Gough and D. Hanson (74) who measured the heat output of the specimen by a calorimetric method.

It is known that adiabatic dilatation decreases the temperature of the specimen, while adiabatic compression increases it. With the initiation of plastic deformation, a high increase of temperature takes place, because more than 90 per cent of the energy input is transformed into heat. Therefore, the yield limit is sharply defined by measuring the heat or temperature. This phenomenon was investigated by C. A. P. Turner (75), E. Rasch (76), R. Plank (77), J. A. Capp and T. R. Lawson (78), H. Hort (79), and G. Tammann (80).

Based on the above phenomena, C. E. Stromeyer (81) developed a method for determining the fatigue limit by a calorimetric method. Up to a certain range of stress, no evolution of heat from the specimen was observed, but beyond this critical stress perceptible heat was generated. The heat generated increased rapidly with stress beyond this critical value. The experiment was carried out on many metals and Stromeyer found that the stress range necessary to produce the given rise in temperature of 0.02° C of the cooling water used in his calorimetric approach was remarkably constant for different specimens of the same material, but varied greatly for
different metals. Stromeyer claimed that the range of stress determined by
the calorimetric method was the fatigue range for the material. This method
of Stromeyer was further developed by H. J. Gough (82). He measured the
change of temperature of the specimen by a thermocouple, as a function of
the stress amplitude. Gough found, in agreement with Stromeyer, that at
a certain stress which is identical with the fatigue limit for a large number
of materials an evolution of heat takes place. After experimenting with more
than eighty materials Gough (83) concluded that the fatigue range as given by
the dynamic proportional limit and the calorimetric methods corresponds
closely with the fatigue limit as determined by the Wohler Method with
relatively few exceptions. For light alloys, for example, this method may
be misleading.

Detailed comparative experiments of this type using a temperature
measurement method were carried out by Moore and Jasper (84). The 48
specimens tested showed deviations from 0 to 4.4 per cent. The fatigue
limit of 25 specimens was lower than the temperature limit; in the rest of
the specimens it was vice versa.

Putnam and Harsch (85) investigated the rise of temperature
resulting from alternating loading by a special apparatus they developed.
They observed two marked points in the temperature-stress curve. The
first point was claimed to be caused by small failures in the material,
while the second point should be associated with the fatigue limit. Some of
these data are shown in Figure 17.

R. Stribek (38) presented data, which appears to be identical
with those of Putnam and Harsch, to arrive at the same conclusion.

Galibourg and Laurent (86) found that the stress of accelerated
temperature rise corresponds to the proportional limit rather than to the
fatigue strength.

E. Lehr (87) performed an experiment of this type using a
rotating beam testing machine he developed. The damping energy was
determined from the torque measured by a simple balance system. He
measured the change of damping as a function of stress and observed two
significant points of the damping energy-stress curve as shown in Figure 18.
One point (A) marks the beginning of deviation of the curve from the straight
line; the second point (B) is obtained from the abscissa intercept of a line
drawn tangent to the higher damping portion of the damping-stress curve.
The damping was measured after about 1500 cycles (one minute after the
beginning of the test) when the value of the damping energy had stabilized according to Lehr. In general, the fatigue limit is near point A for steels with a small initial damping, and near point B for steels with a high damping.

P. Ludwik used, in most cases, the Schenk torsion and bending fatigue testing machine with an optical system for indicating the hysteresis loop. He measured the change in damping during sustained cyclic stress and found, for steel, a maximum in the D-N curve (88). No relationship was found to exist between this maximum or other damping trends and the fatigue strength. Further, the experiments showed, as indicated in Figures 19, 20, 21, and 22, that the damping is not stable in some cases even after $10^8$ cycles. The increase of damping in the first stages was also observed by H. F. Moore and J. B. Kommers (89) and designated as "heat burst".

P. Ludwik and R. Schau (90) made comparative experiments of damping energy as a function of number of cycles on tool steel (StC 110), high carbon steel (StC 100), mild steel, Si-steel, Al-Si-Cu-Mn alloy and an Mg alloy. They compared the change of temperature of the specimen with the total damping energy and investigated also the influence of cooling of the specimen on damping (Figure 23).

Since the temperature increase in uncooled specimens is significant at high stress levels, cooling the specimen so as to maintain room temperature resulted in a significant decrease in damping. They claimed that the temperature rise caused by increasing alternating stresses marks the fatigue limit in the same way as the rise of damping.

P. Ludwik (91) also investigated the difference between the temperature rise and the rise in damping and the dynamic proportional limit. He found that for magnesium and aluminum-copper alloys the point of temperature rise occurred below and the dynamic proportional limit occurred above the fatigue strength. Duralumin showed a smooth D-N curve and no marked point of rise of damping. Pure copper and brass showed the point of rise of temperature and the dynamic proportional limit far below the fatigue limit. Mild steel showed the same, but not as large a deviation. The fatigue strength of Si-steel was identical with the point of temperature rise and the dynamic proportional limit. Soft Cr-Ni steels have the fatigue limit above the point of temperature rise, while hardened and tempered steel showed no significant discontinuity in its damping curve. Cr-Si steels showed a similar behavior to the Cr-Ni...
steels. In general, the dynamic proportional limit was found to be at stresses higher than those producing a significant temperature increase.

W. Herold (92) investigated the damping measurement type of short-time fatigue tests on about 80 different steels. He found the best agreement between the results of his short-time test and the Wohler test for steels with an ultimate strength of about 100,000 psi as shown in Figure 24. Steels with a smaller strength had a high damping, which causes the point of the rise of damping to be significantly below the fatigue strength.

A deviation method employing damping measurements to find the fatigue limit was developed by A. Esau and H. Kortum (93). In this method the alternating stress was raised in steps and cyclic stress was continued until stabilization of the damping occurred. At a definite stress the damping failed to stabilize and steadily increased. The authors claimed the stress to be indicative of the fatigue limit. This method probably has very little applicability since most materials do not appear to display stabilized damping at all stresses below the fatigue limit.

8.3 Damping at Low Stress

E. Gerold and A. Karius (94) investigated the influence of alternating stresses upon the damping at small stresses, and its connection to the fatigue limit. Damping at low stresses as a function of alternating stress magnitude shows, according to these investigators, changes similar to those at high stress. In accordance with Lehr's method, Gerold and Karius determined the fatigue limit from the alternating stress at which low stress damping rises. They examined Brass 58, Al, Al-Cu-Si alloy and steel by this method, and found it only applicable for steels with less than 0.6 percent C. The change of the natural frequency could be measured by the same equipment. It was found that the change of the natural frequency caused by alternating stresses was in accordance with the change of damping (Figure 25).

Since lattice distortions caused by plastic deformations increase the damping, this method is also an indicator of the initiation of plastic deformation. Not only is this method subject to the uncertainties of the Lehr Method but there is also another factor which adds to the total uncertainty. Damping at low stress indicates any kind of change of lattice condition. Besides plastic deformation, phase changes or change of state of solution, etc., also
cause a change of lattice distortion. These effects are superimposed which makes the detection of the fatigue limit difficult. Further research must be done to clarify this problem.

All the short-time fatigue testing methods based on damping measurement as discussed above do not take into account stress history effects. P. Ludwik found that the effect of stress history on damping at stresses below the fatigue strength was small in most cases. B. J. Lazan and T. Wu (56) found that for mild steel damping does not change with stress history until a certain critical value, which they designated as the "cyclic stress sensitivity limit," is reached. At stresses below this limit there seems to be no effect of stress history on damping (See Figure 26). The relationship of the cyclic stress sensitivity limit to the fatigue limit and its possible use in accelerated fatigue testing is currently under study at Minnesota. One advantage in using this limit is that it is independent of stress history during a stepwise load increase test. Cyclic stress sensitivity limit is found to exist not only for mild steel, but recent work at Minnesota indicates that other materials, including alloy steels, gray iron, aluminum alloys, and magnesium alloy, also display this critical stress. In general, the ratio of cyclic stress sensitivity limit to fatigue strength was found to be in the range of 0.75 to 0.90 for the materials investigated.

SECTION IX. CHANGE IN MODULUS OF ELASTICITY

Comparative experiments of B. J. Lazan and T. Wu (56) showed that the tangent modulus of mild steel unlike its damping or proportional limit was not affected by stress history below the fatigue limit. However, cyclic stress above the fatigue limit decreased the tangent modulus progressively with increasing number of cycles (Figure 27). If other materials display the same behavior the change of tangent modulus by stress history might have possible use for rapid indication of fatigue properties.
SECTION X. MAGNETIC PROPERTIES

E. Moench (95) proposed a short-time fatigue test method for ferromagnetic materials. This method is based on the fact that plastic deformation and fatigue changes significantly the magnetic induction of a ferromagnetic material which can be measured by a relatively simple method. Moench found that every ferromagnetic material has its own "magnetic stress limit" which can be obtained in a short-time test. This magnetic stress limit is reasonably close to the fatigue limit for mild steel, but deviates for steel with a higher carbon content. Further experiments on this problem have been carried out by F. Forster and K. Stambke (96), K. Fink and H. Lange (97), H. Fink and W. Hempel (98), and R. L. Cavanagh (99), without giving more details.

SECTION XI. ELECTRICAL RESISTANCE

Shoji Ikeda (100) proposed in 1928 to use the change of electrical resistance caused by alternating stresses for a rapid determination of the fatigue limit. This method was investigated in detail by F. H. Moore and Seichi Konzo (101). The test was started at low stresses and the stresses were gradually increased stepwise. At the end of every stepwise increase in stress, readings of temperature and electric resistance of the specimen were taken. The curve of electric resistance versus stress magnitude shows a definite break at stresses in the range of the fatigue limit. Comparative tests were made by Moore and Konzo with Armco iron, 0.2 carbon steel, 0.52 carbon steel, hardened tool steel, brass, monel metal, and copper (Figure 28). The tests showed a fair coincidence between the fatigue limit as determined by the electric-resistance test and by a regular long-time fatigue test. In general, the electric-resistance test gave results in the "safe" range. The average deviation between the limit as determined by the two methods was 4.67 per cent, and the maximum 9.4 per cent (with the exception of 0.2 carbon steel which had a larger deviation). The electric-resistance tests were in fair coincidence with the rise-of-temperature test. In the case of monel metal the rise-of-temperature test failed while the electric-resistance method gave good results.
Changes in the electric resistance of a material are caused (a) by a change of temperature, and (b) by the structural rearrangement within the metal (state of order of the lattice). In general, the electric resistance of a metal is increased by a change of its structure from the ordered to the disordered state, as is caused, for example, by cold working. Moore and Konzo made the important observation that in some cases the electric resistance decreased at alternating stresses below the fatigue limit, in spite of the rise in temperature (Figure 28). From these results it may be concluded that alternating stresses below the fatigue limit may cause a change of the structure of the metal from the disordered to the ordered stage. This is in accordance with the observations of F. Vitovec (102) that the free energy of low carbon steel is diminished by alternating stresses below the fatigue limit. At stresses beyond the fatigue limit, an analysis must take into account the existence of several zones caused either by stress distribution or surface factors. One zone is at the surface whose lattice distortion is increasing, and a neighboring zone in which the lattice distortion is decreasing (103).

In general, the change of electric resistance caused by cold working is more pronounced than that due to structural rearrangement.

SECTION XII. SURFACE ACTIVITY OF STRESSED MATERIAL

Recently J. Kramer (104) found, that any kind of cold working of metals and some non-metals causes a weak electron radiation. The phenomenon results from the lattice distortions and increase of internal potential energy of the material caused by cold working. Cold working causes a metal to attain a more disordered lattice. As a result of the thermionic motions of the atoms, recovery takes place causing a disordered transformation which is accompanied by the release of electrons. The resultant radiation depends upon magnitude of strain, temperature, time, and surface conditions of the specimen. The change of radiation by cold working and tempering conforms to the change of the electrical resistance. Because of the uncontrolled influence of the surface condition only qualitative measurements can be made at present. Nevertheless, the radiation effect can be used for comparative examinations and may, therefore, have possibilities in materials testing.
Fatigue tests on copper showed an increase of radiation at a definite stress which may be in the range of the fatigue strength (Figure 29). Based on his experiments, Kramer proposed to use the effect of change of radiation by alternating stressing for short-time fatigue tests. Kramer made only a few exploratory tests, therefore extensive experiments are necessary to prove Kramer's proposal.

Change of electrical resistance, coefficient of thermal expansion, and radiation effect are intimately related. It seems that exact comparative investigations of these effects could provide further knowledge regarding the mechanism of fatigue. However, the changes of these effects are very small and, therefore, sensitive and usually complicated equipment is required.

SECTION XIII. EFFECT OF PRIOR FATIGUE STRESS ON ULTIMATE TENSILE STRENGTH

The method of H. F. Moore and H. B. Wishart (105) is based on the theory that below the fatigue limit cycles of repeated stress increase the tensile strength, while above this limit cracks begin to develop and propagate, whereby the tensile strength is reduced. These investigations subjected five or six fatigue specimens to approximately 1,400,000 cycles, each at a different stress. The stresses for the different specimens covered a range of values on both sides of the estimated fatigue strength. After each fatigue test, the tensile strength of the specimens were determined. The alternating stresses were plotted versus the corresponding tensile strengths to determine if the fatigue limit can be predicted from the peaked condition of this curve. This method was tried on structural steel, cold-rolled steel, brass, monel metal, nickel-steel, chromium-nickel steel, duralumin, and specimens of cold-rolled steel with sharp notches. As shown in Figure 30, the short-time test predicted the fatigue limit satisfactorily for all materials except brass and duralumin. These exceptions have also been observed by the other short-time test methods and are not surprising, since change of phase caused by alternating loading during the test takes place, which could not be covered by a short-time test.
However, since the change of the tensile strength caused by alternating stresses is small the resultant flat curve makes it difficult to define a peak value with certainty.

SECTION XIV. COEFFICIENT OF THERMAL EXPANSION

A rapid method for determining the fatigue limit of materials has been recently proposed by J. L. Rosenholtz and D. T. Smith (106) as their "Dilastrain Method". This method depends on the effect of prestressing a specimen by a definite number of cycles upon the coefficient of linear thermal expansion. The coefficient of linear thermal expansion was obtained by heating the specimens from 20° C (68°F) to 100° C (212° F) or from 25° C to 45° C. The number of prestressing cycles for best indication depends upon the type of material and lies in the range of 10,000 to 50,000 cycles for aluminium alloys, brass, and bronze and of 80,000 to 100,000 cycles for steel. The authors present limited data of the type shown in Figure 31 and claim that the fatigue limit coincides with the minimum point as shown. With some alloys the coefficient falls regularly to the fatigue limit and then rises, producing a sharp "V". It should also be noted that the change of the coefficient is very small and in the range of 1 to 5 per cent. Since the temperature range used for coefficient measurement necessarily was relatively small, instruments of great sensitivity are required.

This method must be investigated more thoroughly before it can be accepted.

SECTION XV. DETERMINATION OF THE FATIGUE LIMIT BY X-RAY DIFFRACTION

15.1 Method of F. Regler

F. Regler (107) claims to have observed sharp edges on the back-reflection X-ray lines which enabled him to measure exactly the radial breadth of the reflections. Furthermore, he reports an increasing broadening
of the reflections caused by fatigue stressing. Based on these observations, he proposed methods for determining the fatigue limit and forecasting the life of a specimen which is loaded beyond the fatigue limit. No one other than Regler has reported observation of these sharp edges on the reflections nor reproduced his experiments (108), (109), and (5). It seems that the sharp edges of the reflections observed by Regler were probably Laue asterism resulting from the polychromatic background superimposed on the monochromatic X-rays.

15.2 Method of R. Glocke (110)

R. Glocke used the characteristic radiation of chromium to measure lattice parameter as an indication of surface strains to a depth of about 0.004 inches. At constant alternating loading beyond the fatigue limit, he observed a continuous decrease in surface strain. In most cases, no decrease could be observed at loadings below the fatigue limit. However, if an initial decrease did take place below the fatigue limit it was followed by an increase caused by strain hardening as shown in Figure 32. R. Glocke proposed to use this effect for rapid determination of the fatigue limit because only 200,000 cycles are necessary for detection, whether the loading is in the safe range or not. This method was tried by J. A. Bennett (111). He found that a stress smaller than the fatigue strength produces a decrease of the surface strain in the same way as stresses beyond the fatigue strength.

SECTION XVI. STRENGTH UNDER COMBINATION OF ALTERNATING BENDING AND TENSION LOADS

E. Mohr (112) developed a method for rapid determination of the fatigue limit primarily for thin sheets and wires. Each specimen was subjected to constant tensile force and to a fixed alternating bending angle causing bending strain considerably smaller than the tensile strain. To establish the S-N curve different specimens were subjected to the same cyclic bending angle, but each specimen had a different tensile force as indicated in Figure 33. Mohr claims that the stress at the break in his curves is identical with the fatigue limit of steel and with the fatigue strength of non-ferrous metals at $20 \times 10^6$ cycles.
F. Erdmann-Jessnitzer, H. Hanemann, and E. J. Kohlmeyer (113) found no timesaving by this method for testing Zn and Zn-alloys.

SECTION XVII. SUMMARY AND ANALYSIS OF METHODS

Since fatigue cracks are, in general, brittle tensile cracks, proportionality between fatigue strength and tensile strength was assumed in early work. However, no general relationship of this type could be found for all types of materials and all conditions. The relationship between fatigue and other static properties such as proportional limit, yield strength, and true tensile strength have been considered again without success. This approach has been elaborated upon by developing formulas, particularly for steel, which give the fatigue limit as a function of several static properties such as yield strength, apparent tensile strength, elongation, and reduction of area, etc. These formulas seem applicable only under special and highly limited conditions. However, these formulas have been used rather widely in the steel industry of Germany.

Based on the fact that fatigue is caused by reversed slipping, the fatigue limit was proposed to be identical to that stress at which slip lines begin to form or at which slip lines do not appear again after prestressing. No proportionality between this so determined stress and the fatigue strength could be observed since other secondary effects such as strain hardening, aging, etc., influences the fatigue properties.

Another attempt for reducing fatigue testing time is based on assumptions that the shape of the S-N curve is fixed. Assuming the S-N curve to be a hyperbola, two or three reliable points may be sufficient for mathematical or graphical determination of the fatigue limit. The Prot method also must use this assumption, among others, to interpret the fatigue strength under uniformly increasing load. The validity of the methods based on this assumption seems to be very limited since numerous fatigue tests show that S-N curves frequently do not display a hyperbolic shape.
Contrails

Attempts have also been made to associate fatigue properties with the stress-strain characteristics under reversed stress. A large number of fatigue tests showed that the dynamic proportional limit gives a good indication of the fatigue strength for many metals and alloys and appears to have fewer exceptions (for example, duralumin) than do other methods.

In several other methods the change of other physical properties caused by alternating stress have been investigated for possible association with fatigue properties. Properties studied in this way include damping, magnetic properties, electrical resistance, coefficient of thermal expansion, mosaic size detected by X-rays, surface stresses detected by X-rays, surface activity, and ultimate tensile strength. In general the change of the property as a function of reversed stress only has been investigated, and only recently have stress history effects been studied. All of these physical properties have been found to be affected by fatigue stress, but in most cases the magnitude of change is relatively small and therefore difficult to determine accurately. To date, insufficient basic work has been completed to clarify the significance of such associations.

In other groups of short-time tests fatigue rupture properties are determined under conditions of uniformly increasing stress or other types of constant load condition. Special attention may be directed to Prot's method in which the stress is uniformly increased until failure. For reasons discussed previously the progressive load increase method does not appear to be applicable for all materials.

Generally speaking, some of the short-time fatigue tests seem to be very useful as comparative tests. No one or few short-time tests display definite superiority over the other methods and the best test for a given project depends on individual circumstances and objectives. For any new material or material in such condition where the fatigue properties are unknown, short-time tests will provide only an approximate indication of fatigue properties; regular long-time tests must be used if accurate fatigue properties are required.
BIBLIOGRAPHY


52. D. C. McAdam, Jr.: Accelerated Fatigue Tests and Some Endurance Properties of Steel. Their Relation to Other Physical Properties and to Chemical Composition. ASTM 23 (1923) p. 56.


54. J. M. Lessels: Endurance Properties of Steel, the Relation to Other Physical Properties and to Chemical Composition. ASTM 23 (1923) II.


FIG. 2 - FATIGUE LIMIT VERSUS TENSILE STRENGTH OF SEVERAL STEELS (E. HOUDREMONT & R. MAILAENDER).

FIG. 3 - FATIGUE LIMIT VERSUS TENSILE STRENGTH OF STEEL (BATTELLE MEMORIAL). WADC TR 53-122
**FIG. 4** - FATIGUE LIMIT VERSUS TRUE TENSILE STRENGTH BASED ON NECKED AREA (E. HEROLD).

**FIG. 5** - FREQUENCY DISTRIBUTION OF THE FLUCTUATIONS OBTAINED FROM DIFFERENT FORMULAS FOR THE DETERMINATION OF THE FATIGUE LIMIT FROM THE TENSILE PROPERTIES OF STEEL (R. MAILAENDER)

**FIG. 6** - FLUCTUATIONS OBTAINED FROM DIFFERENT FORMULAS FOR THE DETERMINATION OF THE FATIGUE LIMIT FROM THE TENSILE PROPERTIES OF STEEL (LEQUIS, BUCHHOLZ, SCHULZ).
FIG. 7 - DEVIATION OF THE FATIGUE LIMIT CALCULATED FROM DIFFERENT FORMULAS FROM THE ORIGIN (R. MAILAENDER).

FIG. 8 - YIELD STRENGTH VERSUS ULTIMATE STRENGTH FOR SEVERAL STEELS (E. HOUDREMONT & R. MAILAENDER).

FIG. 9 - RELATIONSHIP BETWEEN FATIGUE LIMIT AND TENSILE STRENGTH OF STEEL WITH DIFFERENT RATIOS OF YIELD STRENGTH TO TENSILE STRENGTH (A. FRY, A. KESSNER, AND R. OETTEL).
FIG. 10 - FATIGUE LIMIT VERSUS RATIO YIELD LIMIT/ULTIMATE STRENGTH OF SEVERAL STEELS (M. HEMPEL AND H. KRUG).
FIG. 11 - FAILURE STRESS VERSUS DESTRUCTION VELOCITY OF STEEL (W. SPAETH).

FIG. 12 - MEAN STRESS VERSUS MEAN STRAIN UNDER CONSTANT CYCLIC STRESS BELOW THE FATIGUE LIMIT (T. H. SMITH).
FIG. 13 - ALTERNATING STRESS VERSUS ALTERNATING STRAIN UNDER PROGRESSIVELY INCREASING LOAD (H. J. Gough).

FIG. 14 - STATIC AND DYNAMIC PROPORTIONAL LIMIT VERSUS NUMBER OF CYCLES OF MILD STEEL (B. J. Lazan and T. Wu).

WADC TR 53-122
FIG. 15—FAILURE STRESS VERSUS SQUARE ROOT OF LOADING RATE OF SAE 4340 STEEL (E. J. WARD AND D. C. SCHWARTZ).
FIG. 16 - FAILURE STRESS VERSUS SQUARE ROOT OF LOADING RATE
OF VARIOUS MATERIALS. STEPWISE LOAD INCREASE (SINCLAIR).
WADC TR 53-122
FIG. 17 - SPECIMEN TEMPERATURE VERSUS ALTERNATING STRESS FOR DIFFERENT KINDS OF STEEL (PUTNAM AND HARSCH).

FIG. 18 - DAMPING ENERGY VERSUS MAGNITUDE OF ALTERNATING STRESS (E. LEHR).

WADC TR 53-122

FIG. 20 – TEMPERATURE RISE OF THE SPECIMEN VERSUS NUMBER OF CYCLES AT DIFFERENT CONSTANT STRESSES. CARBON STEEL WITH 68,300 PSI. ULTIMATE STRENGTH (P. LUDWIK).

WADC TR 53-122
FIG. 21 - DAMPING ENERGY VERSUS NUMBER OF CYCLES AT CONSTANT SHEAR STRESS. FATIGUE STRENGTH 19,200 PSI. (P. LUDWIK AND R. SCHEU).

FIG. 22 - TEMPERATURE RISE OF THE SPECIMEN VERSUS NUMBER OF CYCLES AT THE FATIGUE LIMIT OF STEELS WITH DIFFERENT CARBON CONTENT (P. LUDWIK).
FIG. 25 - LOGARITHMIC DECREMENTS AND NATURAL FREQUENCY VERSUS ALTERNATING STRESSIE, GEROLD B. KARIUS.

FIG. 24 - FREQUENCY OF DEFINITE DEVIATION BETWEEN STRENGTH BASED ON DAMPING TESTS AND WOHLER FATIGUE STRENGTH AS A FUNCTION OF THE ULTIMATE STRENGTH OF STEEL (W. HEROLD).
FIG. 26 - EFFECT OF REVERSED STRESS MAGNITUDE AND STRESS HISTORY ON DAMPING ENERGY OF STRESS-RELIEVED STEEL (B. J. LAZAN AND T. WU).
<table>
<thead>
<tr>
<th>SPECIMEN</th>
<th>PA 112</th>
<th>PA 196</th>
<th>PA 108</th>
<th>PA 244</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAXIMUM REVERSED STRESS</td>
<td>29,900</td>
<td>31,100</td>
<td>35,600</td>
<td>38,000 PSI</td>
</tr>
<tr>
<td>RUN DESCRIPTION</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>PRIOR CYCLES AT MAX. STRESS</td>
<td>20</td>
<td>30 x 10^6</td>
<td>20</td>
<td>1400</td>
</tr>
<tr>
<td>TANGENT MODULUS, 10^6 PSI</td>
<td>29.4</td>
<td>29.4</td>
<td>29.4</td>
<td>29.4</td>
</tr>
<tr>
<td>PLASTIC STRAIN, P, 10^{-4} IN. PER IN.</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>STATIC PROP LIMIT, Q, PSI</td>
<td>&gt; 29,900</td>
<td>&gt;31,000</td>
<td>25,000</td>
<td>19,000</td>
</tr>
<tr>
<td>DYNAMIC PROP LIMIT, Q', PSI</td>
<td>&gt; 29,900</td>
<td>&gt;31,000</td>
<td>27,000</td>
<td>21,000</td>
</tr>
</tbody>
</table>

**FIG. 27 - EFFECT OF STRESS HISTORY ON STATIC AND DYNAMIC STRESS-STRAIN CURVES FOR MILD STEEL (B. J. LAZAN AND T. WU.)**
FIG. 28 - ELECTRICAL RESISTANCE AND TEMPERATURE OF THE SPECIMEN VERSUS ALTERNATING STRESS FOR MONEL-METAL AND COPPER (F. H. MOORE AND S. KONZO).
FIG. 29 - RATE OF ELECTRON RADIATION VERSUS ALTERNATING STRESS (J. KRAMER).

FIG. 30 - EFFECT OF 1,400,000 CYCLES OF REVERSED FLEXURAL STRESS ON ULTIMATE TENSILE STRENGTH (H. F. MOORE & H. B. WISHART).
FIG. 31 - COEFFICIENT OF THERMAL EXPANSION VERSUS ALTERNATING STRESS (J. L. ROSENHOLTZ AND D. T. SMITH).

FIG. 32 - SURFACE STRESS OF STEEL DETERMINED BY X-RAYS VERSUS NUMBER OF CYCLES AT DIFFERENT ALTERNATING STRESSES.
FIG. 33 - S-N CURVE UNDER CONSTANT BENDING STRAIN AND VARIABLE STATIC TENSILE STRESS (E. MOHR).