DYNAMIC SYSTEM STUDIES:
AERODYNAMIC STUDIES

M. SAARLAS
UNIVERSITY OF CHICAGO

M. Z. KRZYWOBLOCKI
UNIVERSITY OF ILLINOIS

SEPTEMBER 1956

WRIGHT AIR DEVELOPMENT CENTER
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SEPTEMBER 1956

AERONAUTICAL RESEARCH LABORATORY
PROJECT 7060
ADVISORY BOARD ON SIMULATION
CONTRACT No. AF 33(038)-15068, SUPPLEMENTS 2 AND 11

WRIGHT AIR DEVELOPMENT CENTER
AIR RESEARCH AND DEVELOPMENT COMMAND
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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The Advisory Board on Simulation has concluded a three-year research program in air weapon system dynamics sponsored by Wright Air Development Center, with P. W. Nosker/WCRR as project engineer. This volume is one of the following 16 comprising the final report, WADC TR 54-250, entitled **Dynamic System Studies**:

<table>
<thead>
<tr>
<th>Part No.</th>
<th>Subtitle</th>
<th>Editing Agency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Conclusions and Recommendations</td>
<td>University of Chicago</td>
</tr>
<tr>
<td>2</td>
<td>The Design of a Facility</td>
<td>&quot;</td>
</tr>
<tr>
<td>3</td>
<td>The Mission of a Facility (Confidential)</td>
<td>&quot;</td>
</tr>
<tr>
<td>4</td>
<td>Technical Staff Requirements</td>
<td>&quot;</td>
</tr>
<tr>
<td>5</td>
<td>Analog Computation</td>
<td>&quot;</td>
</tr>
<tr>
<td>6</td>
<td>Operation &amp; Maintenance Procedures for Analog Computers</td>
<td>&quot;</td>
</tr>
<tr>
<td>7</td>
<td>Digital Computers</td>
<td>&quot;</td>
</tr>
<tr>
<td>8</td>
<td>Recorders</td>
<td>&quot;</td>
</tr>
<tr>
<td>9</td>
<td>Flight Tables (Confidential)</td>
<td>&quot;</td>
</tr>
<tr>
<td>10</td>
<td>Performance Requirements for Flight Tables</td>
<td>Mass. Inst. of Tech.</td>
</tr>
<tr>
<td>11</td>
<td>Load Simulators (Confidential)</td>
<td>Cook Research Lab.</td>
</tr>
<tr>
<td>12</td>
<td>Guidance Simulation (Secret)</td>
<td>Naval Ord. Lab., Corona</td>
</tr>
<tr>
<td>13</td>
<td>Error Studies</td>
<td>University of Chicago</td>
</tr>
<tr>
<td>14</td>
<td>Error Analysis for Differential Analyzers (written by F.J. Murray, Columbia U., and K.S. Miller, N.Y.U.)</td>
<td>&quot;</td>
</tr>
<tr>
<td>15</td>
<td>Air Vehicle Characteristics (Secret)</td>
<td>&quot;</td>
</tr>
<tr>
<td>16</td>
<td>Aerodynamic Studies (written by M.Z. Krzywoblocki, U. of Ill.)</td>
<td>&quot;</td>
</tr>
</tbody>
</table>

The history of the project and a complete bibliography may be found in Part 1. All reports may be obtained through the project engineer.

This report represents the culmination of the assignment to determine the proper mission, equipmentation, operation procedures, and personnel for an engineering facility in the field of air weapon systems dynamics. The sub-
divisions of the report correspond to these four basic objectives and the subsidiary work in their support, and reflect the role of simulation as a dominant technique. The function of each part and the relations among them are indicated in the technical summary, Part 2.

The following organizations have participated directly in the program:

<table>
<thead>
<tr>
<th>Organization</th>
<th>Contract No.</th>
<th>Time of Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>University of Chicago</td>
<td>AF33(038)-15068 Supplements 2 and 11</td>
<td>1 Feb. '51-31 Aug. '54</td>
</tr>
<tr>
<td>J. B. Rea Company</td>
<td>AF33(038)-15068 Subcontract 2</td>
<td>1 Feb. '51-31 Oct. '52</td>
</tr>
<tr>
<td>Cook Research Laboratories</td>
<td>AF33(038)-15068 Subcontracts 3 and 9</td>
<td>1 Feb. '51-31 May '54</td>
</tr>
<tr>
<td>RCA Laboratories</td>
<td>AF33(038)-15068 Subcontract 4</td>
<td>1 Feb. '51-1 Mar. '53</td>
</tr>
<tr>
<td>Armour Res. Foundation of Ill. Inst. of Technology</td>
<td>AF33(038)-15068 Subcontract 5</td>
<td>1 Feb. '52-30 Nov. '52</td>
</tr>
<tr>
<td>Northwestern University Aerial Meas. Lab.</td>
<td>AF33(038)-15068 Subcontract 8</td>
<td>17 July '52-22 Aug. '52</td>
</tr>
<tr>
<td>Mass. Inst. of Technology, Flight Control Lab.</td>
<td>AF33(038)-15068 Purchase Order A2086</td>
<td>20 Apr. '54-31 Aug. '54</td>
</tr>
<tr>
<td>Mass. Inst. of Technology, Dynamic Analysis and Control Laboratory</td>
<td>AF33(038)-15068 Purchase Order A23883</td>
<td>22 July '53-30 Nov. '53</td>
</tr>
<tr>
<td>Nat. Bur. of Standards Corona, which became</td>
<td>AF33(038)-51-4345-E</td>
<td>25 Feb. '51-Sept. '53</td>
</tr>
<tr>
<td>Naval Ordnance Lab., Corona</td>
<td>MIPR(33-616)54-154</td>
<td>20 Nov. '53-31 Dec. '55</td>
</tr>
</tbody>
</table>

This is a record of formal participation only; the program was aided immeasurably by the splendid cooperation of all governmental, industrial, and educational organizations (particularly the simulation laboratories) contacted. Although it is impractical to mention them all here, the extent of their assistance is evident throughout the reports and is hereby gratefully acknowledged. Details of these affiliations, including statements of work, may be found throughout the 21 Bimonthly Progress Reports issued by the University of Chicago during the course of the work. (All formal participation in the program is recorded above;
missing supplement and subcontract numbers do not pertain to this project.)

The University of Chicago was assigned prime responsibility for integration of the program. This has been effected by a full time staff at the University, and by aperiodic meetings of the following advisory committee, selected by the Air Force:

Dean Walter Bartky, Chairman  University of Chicago 1 Feb. '51-31 Aug. '54
Prof. C. S. Draper  Mass. Inst. of Tech. 1 Feb. '51-28 Feb. '53
Mr. Donald McDonald  Cook Research Lab. 1 Feb. '51-31 Aug. '54
Prof. F. J. Murray  Columbia University 1 Apr. '52-31 Aug. '54
Dr. J. B. Rea  J. B. Rea Company 1 Feb. '51-28 Feb. '53
Prof. R. C. Seamans, Jr.  Mass. Inst. of Tech. 1 Sept. '53-31 Aug. '54
Mr. R. J. Shank  Hughes Aircraft Co. 1 July '51-31 Aug. '54
Dr. H. K. Skramstad  NBS-NOLC 1 Feb. '51-31 Aug. '54
Mr. A. W. Vance  RCA Laboratories 1 Feb. '51-31 Aug. '54

ex officio:
Mr. P. W. Nosker, Project Eng. WADC 1 Feb. '51-31 Aug. '54
Dr. B. E. Howard, Secretary  University of Chicago 1 Feb. '51-31 Aug. '54

The meetings have been recorded in the Bimonthly Progress Reports previously mentioned. Except for Dr. Skramstad, who has participated through direct arrangement between NBS-NOLC and WADC, members of the advisory committee who are not connected directly with the University have participated in the program through consulting agreements with the University of Chicago. In addition, similar consulting agreements with the University have provided for the participation of:

Dr. R. R. Bennett  Hughes Aircraft Co. 1 Jan. '52-31 Jan. '54
Mr. J. P. Corbett  Libertyville, Ill. 11 May '54-31 Aug. '54
(formerly with the University)

Mr. G. L. Landsman  Motorola, Inc. 1 May '54-31 Aug. '54
Dr. Thornton Page  Johns Hopkins Univ. 7 Aug. '51-1 Mar. '53
(formerly with the University, and Secretary to the Board until 1 Aug. '51)
Many others have contributed significantly to the progress of the work. Among those from other organizations in regular attendance at most of the meetings of the committee have been Mr. Charles F. West, Air Force Missile Test Center; Prof. L. L. Rauch, University of Michigan, representing Arnold Engineering Development Center; Col. A. I. Lingard, WADC; and Dr. F. W. Bubb, WADC.

Coordination of the program and administration of the prime contract at the University of Chicago have been under the charge of Dr. Walter Bartky, Dean of the Division of Physical Sciences and Director of the Institute for Air Weapons Research; Dr. B. E. Howard, Assistant to the Director; and Messrs. William R. Allen and William J. Riordan, Group Leaders. The work at the cooperating institutions has been directed by the appropriate member of the advisory committee and his assistants: Dr. H. K. Skramstad and Mr. Gerald L. Landsman at the National Bureau of Standards-Naval Ordnance Laboratory, Corona; Messrs. Donald McDonald and Jay Warshawsky at Cook Research Laboratories; Messrs. A. W. Vance, J. Lehman, and Dr. E. C. Hutter at RCA Laboratories; Dr. J. B. Rea at J. B. Rea Company; Prof. R. C. Seamans at the Flight Control Laboratory and Dr. W. W. Seifert and Mr. H. E. Blanton at the Dynamic Analysis and Control Laboratory, Mass. Inst. of Technology. V. H. Disney, S. Hori, and G. F. Warnke at Armour Research Foundation and J. C. MacAnulty and George Goelz at Northwestern University, Aerial Measurements Lab., have directed the contributory studies at their respective organizations. More explicit credit is found in appropriate places throughout the reports; biographical sketches are in Part 1. Space does not allow full credit that is due to all the workers on the combined project, but special mention is certainly due the project engineer for his conception of the project and for his cooperation during its execution.
This part of the report has been written by Maida Saarlas of the University of Chicago and M. Z. Krzywoblocki of the University of Illinois, Mr. Saarlas having prime responsibility for chapters 1 through 4, Dr. Krzywoblocki for chapters 5 and 6. Conception, plan, and organization of the volume are largely the work of Dr. B. E. Howard, who has also aided materially in writing many sections. The volume was prepared for publication under the direction of E. R. Spangler at the University of Chicago.
ABSTRACT

The trajectory of an air vehicle is determined by Newton's laws; it is necessary to know the aerodynamic forces to solve $F = ma$. The forces are composed of body forces (gravity and thrust) and the surface forces (air reactions) which depend on (1) kinematics (linear and angular velocities $u, v, w, p, q, r$), (2) geometry (body shape and control surface deflections $\delta_a, \delta_e, \delta_r$), and (3) properties of the medium (density, pressure, temperature, viscosity, heat conductivity). No general expressions for the forces are available. General functional relations and the influence of the independent variables on the force and moment components are discussed. The usual engineering approach in designing airplanes involves evolution of existing models based on empirical formulae and experience. Expressions used in simulation are based on Taylor series and test measurements. It is felt impractical and unnecessary to refine algebraic expressions for the forces. Present efforts are directed toward the electronic solution of the differential equations giving rise to the forces. The goal is a mechanization of the appropriate equations of fluid motion, continuity, energy and state such as to provide continuous D.C. voltages representing the force and moment components, without the necessity of having exact explicit expressions for them. Modern trends in fluid dynamics relevant to this goal are discussed.

PUBLICATION REVIEW

The publication of this report does not constitute approval by the Air Force of the findings or the conclusions contained therein. It is published only for the exchange and stimulation of ideas.

FOR THE COMMANDER:

ALDRO LINGARD
Colonel, USAF
Chief, Aeronautical Research Laboratory
Directorate of Research

WADC TR 54-250, Part 16
<table>
<thead>
<tr>
<th>TABLE OF CONTENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreword i</td>
</tr>
<tr>
<td>Abstract vi</td>
</tr>
<tr>
<td>Introduction ix</td>
</tr>
<tr>
<td>1. The Motion of the Aircraft 1</td>
</tr>
<tr>
<td>1.1 The Flight Path 1</td>
</tr>
<tr>
<td>1.2 Source of Equations and Variables 1</td>
</tr>
<tr>
<td>1.3 The Difficulties in Obtaining Exact Expressions 2</td>
</tr>
<tr>
<td>1.4 Information Necessary for Simulation 2</td>
</tr>
<tr>
<td>2. The Forces and Moments 4</td>
</tr>
<tr>
<td>2.1 The Variables and Symmetry Considerations 4</td>
</tr>
<tr>
<td>2.2 Functional Relationships 4</td>
</tr>
<tr>
<td>2.3 Influence of Separate Variables 5</td>
</tr>
<tr>
<td>2.4 Quantitative Results 13</td>
</tr>
<tr>
<td>3. Classical Aeronautical Engineering 14</td>
</tr>
<tr>
<td>3.1 The Engineering Approach 14</td>
</tr>
<tr>
<td>3.2 The Assumptions 14</td>
</tr>
<tr>
<td>3.3 Step-by-Step Integration 16</td>
</tr>
<tr>
<td>3.4 The Separate Airplane Sections 16</td>
</tr>
<tr>
<td>3.5 Present Equations 17</td>
</tr>
<tr>
<td>4. The Engineering Approach and Simulation 23</td>
</tr>
<tr>
<td>4.1 The Problem: Expressions for Forces and Moments 23</td>
</tr>
<tr>
<td>4.2 Classical Considerations 23</td>
</tr>
<tr>
<td>4.3 Simulation vs. Aeronautical Engineering 24</td>
</tr>
<tr>
<td>4.4 A New Method 25</td>
</tr>
<tr>
<td>5. The Modern Trend in Fluid Dynamics, Aerodynamics, Gasdynamics, and Thermoaerodynamics 27</td>
</tr>
<tr>
<td>5.1 Fundamentals of Fluid Dynamics 27</td>
</tr>
<tr>
<td>5.2 The Engineering Approach 28</td>
</tr>
</tbody>
</table>
Dynamic system engineers today are confronted with an extremely difficult task: in the face of increasing complexity of the systems and more rigid performance specifications, to predict the behavior of an air vehicle in flight. Experience has proved that the classical methods of mathematics fail and that different tools have to be employed. The Air Force has recognized this need and for this reason has initiated the program aimed at establishing a dynamic system laboratory, the principal tool of which is simulation.

The object of simulation is to determine the behavior of an air vehicle system by synthetic laboratory devices, without the necessary presence of the whole system itself. This can be achieved by mathematical or physical simulation on specially designed computing devices wired in such a way that at certain points the voltages represent some desired mathematical expressions. Simulation is accomplished if the mathematical behavior of one system can be inferred from the mathematical behavior of the other.

In the given case the fundamental problem is to find the behavior of an air vehicle in flight. This means that it is necessary to find the space curve traced by the center of mass of that air vehicle and the attitude of the vehicle. To do this, it is necessary to define and to determine the forces and the moments acting on the system (the air vehicle) and solve the six equations of motion. As is already well known, this problem does not admit a direct theoretical attack for the present day air vehicle, since no exact and fundamental method for calculating the aerodynamic forces and moments on such a body has yet been evolved. The problem is as old as ballistics itself and nothing new has been presented herein.

The following chapters attempt to give some insight into

(a) what is presently known about the problem
(b) why the explicit expressions for the forces are unknown
(c) what is being done to supply practical knowledge needed for simulation.

Treatment is not exhaustive here. Only the most fundamental principles and problems are discussed, since detailed treatment would involve the very basis of aeronautics and simulation and will require many other volumes. Several
works have already been issued as literature reviews, which serve to keep abreast of current developments in the engineering science, and help to concentrate on defining the force expressions.
1. THE MOTION OF THE AIRCRAFT

The subject matter here covers everything which will fly—conventional aircraft, guided missiles, etc. Some of the later designs exhibit phenomena not encountered in earlier designs, i.e., the Magnus force on a rotating missile. However, the ensuing discussion is in terms of conventional aircraft for the sake of simplicity and clarity.

1.1 The Flight Path

Considering an airplane as a rigid body flying through some medium, its flight path is determined by the earth's gravitational field, the propulsive forces, the airplane's inertia characteristics, and the aerodynamic forces and moments resulting from the reaction between it and the medium through which it moves. Obviously, in order to simulate the motion of the airplane, a thorough understanding of the nature of the forces acting on the airplane is needed.

1.2 Source of Equations and Variables

Since the equations of motion are developed by the application of Newton's laws for each of the airplane's degrees of freedom, in order to solve these equations, expressions are needed to describe the forces. However, at the present time there exist no exact general mathematical expressions for the forces, except perhaps in very special cases. Furthermore, the relationships between the forces and the variables on which they depend are complicated to the extent that there is no way in which to express the forces explicitly in terms of any of those variables. In practice, the problem consists in determining acceptable approximations to these relationships between the forces and the variables.

For a given airplane shape and known properties of the medium (assume air) through which it moves, the air reactions $F_x$, $F_y$, $F_z$, $L$, $M$, $N$ depend upon the motion of the airplane relative to the air. The air reactions also depend on shape. The effective shape of the aircraft can be changed in flight by moving the control surfaces: aileron, rudder, elevator. Hence $F_x$, ..., $N$ depend upon the nine variables $u$, $v$, $w$, $p$, $q$, $r$, $\delta_a$, $\delta_e$, $\delta_r$.

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WADC TR 54-250, Part 16
where \( u, v, w \) = velocity components in \( x, y, z \) directions respectively

\( p, q, r \) = angular velocities around \( x, y, z \) axes respectively

\( \delta_a, \delta_e, \delta_r \) = aileron, elevator, and rudder deflections respectively

and upon their time rates of change.

1.3 The Difficulties in Obtaining Exact Expressions

Up to the present time little, if anything, has been achieved in finding the air reactions for the general case of the airplane motion. As pointed out in what follows, this is due both to:

(a) mathematical difficulties in solving differential equations and
(b) physical difficulties, in the sense that several phenomena are not clearly understood and hence adequate qualitative description is not available.

Some of the difficulties encountered in arriving at an adequate description of the forces are:

(1) All the independent and dependent variables that make up these forces have not yet been completely determined.
(2) How these variables influence the forces is not completely known.
(3) The implicit relationships make a detailed analysis complicated.
(4) The aerodynamic forces, in general, depend on the whole history of the motion of the airplane, which is contradicted by the often-assumed steady-state concept.
(5) As mentioned in the Introduction, no general solution has been found yet for the case when the body moving through the air is an airplane.

1.4 Information Necessary for Simulation

Consequently for effective simulation the following questions need to be answered:

(1) What are the most general and most reliable expressions today for the aerodynamic forces on the air vehicle?
(2) What are the fundamental hypotheses behind these expressions and the inherent limitations on them?

(3) What are the boundaries of accuracy for forces calculated by present methods?

Throughout these first 50 years of flight, engineers have been unable to find completely satisfactory answers to these questions. However, ingenuity and experience have produced a number of approximation methods that have been applied in the design of the present day aircraft, and also partly answer the questions. But for simulation purposes the questions must be answered fully, because incomplete knowledge of the forces acting on an airplane will render the simulation ineffective.

Therefore a discussion of the forces and variables is presented. This discussion will not answer these questions; rather it will attempt to show why the questions have not been answered previously and what is being done to achieve the goal of a practical approach to air reactions.
2. THE FORCES AND MOMENTS

2.1 The Variables and Symmetry Considerations

From the airplane kinematics, in general, the forces and moments are functions of the following variables:

$$u, v, w, p, q, r, \delta_a, \delta_e, \delta_r, a, z_e, M$$

where

- $a$ = angle of attack
- $z_e$ = altitude parameter
- $M$ = Mach number

Strictly speaking, the basic physical variables are pressure, density, and temperature. The forces and the velocity of sound both depend on these quantities (which vary with altitude in a time changing way). It is common to assume a "standard" atmosphere in which $p$, $\rho$, and $T$ are prescribed functions of altitude, to state the velocity of sound as a separate quantity, and to normalize velocity with respect to velocity of sound (Mach number). Thus altitude and Mach number are the variables used instead of $p$, $\rho$, and $T$.

As already mentioned, not much is known about the exact relationships between the forces and these variables. But there are certain tools which permit some qualitative remarks about these relationships. One such tool is the fact that the airplane, in general, is symmetric about its $x$-$z$ plane. As usual in aeronautical engineering, some restrictions arise, from the assumption that the airplane is completely symmetric. This, however, is seldom strictly true, since the following effects tend to destroy the symmetry:

1. minor differences in the mass distribution between the wings (e.g., emptying of the fuel tanks)
2. asymmetrical power effects
3. gyroscopic effects
4. aileron and rudder deflections.

2.2 Functional Relationships

According to the assumption of complete symmetry, the aerodynamic forces and moments referred to body axes (i.e., exclusive of terms introduced by
coordinate transformations) are found to be functions of the following variables:

\[
F_x', F_z', M = f \left( u, w, q, \delta_e, a, z_e, \text{Mach no.} \right) g (v, p, r, \delta_a, \delta_r) \\
F_y, L, N = f (v, p, r, \delta_a, \delta_r, a, z_e, \text{Mach no.})
\]

(2)

where the subscripts on the parentheses signify

- \(g\) = general function
- \(e\) = even function
- \(o\) = odd function

and \(x, y, z\) designate Cartesian coordinates, referred to body axes.

This report discusses aircraft motion with respect to body axes, since this method is more convenient for simulation. If any N.A.C.A. data is used for simulation purposes, it should be transformed to the principal axes system by the method given in reference 3. However, for 95% of the flight time, the difference between the body and wind axes is a very small angle, and that difference can be generally neglected.

Obviously for large deviations from symmetry (large aileron or rudder deflection, failure of one power plant in multi-engine aircraft, etc.) the relationships of (2) do not represent the true physical situation. The forces and moments behave then according to the following general relationships:

\[
F_x = F_x \left( u, v, w, \delta_a, \delta_e, \delta_r, a, z_e, M \right) \\
F_y = F_y \left( v, p, r, \delta_a, \delta_r, a, z_e, M \right) \\
F_z = F_z \left( v, p, r, \delta_a, \delta_r, a, z_e, M \right) \\
L = L \left( v, p, r, \delta_a, \delta_r, a, z_e, M \right) \\
M = M \left( v, p, r, \delta_a, \delta_r, a, z_e, M \right) \\
N = N \left( v, p, r, \delta_a, \delta_r, a, z_e, M \right)
\]

2.3 Influence of Separate Variables

Next, we can investigate separately the influence of a single variable on each force and moment. This method neglects the interaction and the influence of one variable on the other, but it allows a general qualitative discussion of
the forces and the moments.

Essentially the following forces act on the airplane:

1. Body forces: gravity and thrust
2. Surface forces: lift and drag which depend on
   (a) kinematics of motion: \( u, v, w, p, q, r \) and their derivatives; angle of attack as a parameter.
   (b) properties of the medium
      (i) thermodynamic: \( p, \rho, T \); velocity of sound (derived). In theory of flight altitude is commonly used as a fundamental variable with assumed standard atmosphere
      (ii) chemico-physical: coefficient of viscosity, \( \mu \), coefficient of thermal conductivity, \( \lambda \)
      (iii) electromagnetic properties
      (iv) elastic properties
   (c) geometry of the body: shape, size, control surface properties.

(1) is known directly. Total surface forces (2) are obtained in the following fashion:

\[
\int_{\text{over body}} \text{surface force (pressure)} + \int_{\text{over body}} \text{friction} = \text{lift} + \text{drag}
\]

where the first term is the normal force (pressure) and the second force is tangential component (friction = \( \mu V_j \), where \( V \) = velocity vector and comma indicates partial differentiation with respect to \( j^{\text{th}} \) axis.)

\( \rho, T \) and \( V \) are obtained by solving the following six basic flow equations:

3 equations of motion (variables \( u, v, w, \rho, p, \mu \))
1 equation of continuity (variables \( \rho, u, v, w \))
1 equation of energy (variables \( T, p, \lambda, u, v, w \))
1 equation of state (variables \( p, T, \rho \))
Because of the inherent relationship between the variables and equations, the proper procedure is to evaluate $V(u, v, w)$ from the equations of motion and the pressure $p$ from the equation of state. The equations of continuity and energy will be solved for density $\rho$ and temperature $T$, respectively.

Concerning the flow equation of motion, only some special cases have been treated analytically for three degrees of freedom. Since simulation is interested in six degrees of freedom (included $p$, $q$, $v$) new concepts and new equations are needed that are adequate to describe the complete motion of the flow.

2.3.1. Velocity. Considering velocity ($V \approx u$), $F_x$ relationships first, it is a good approximation to write $F_x \approx V^2$. This is evident from the following considerations. Essentially $F_x$ is the difference between the thrust and drag forces. Now the thrust and the drag are, in general, both functions of velocity (the general relationships given in figures 1 and 2). Simple investigation of these phenomena reveals that $F_x$ increases approximately with $V^2$. Determination of the drag on the aircraft is a major problem. The usual procedure is to break the total drag into its components and to evaluate these drag components separately. A rather thorough discussion of the way in which the total drag
can be evaluated in Aerodynamic Studies: "The Forces Acting on an Air Vehicle," Volume I, by M. Z. Krzywoblocki. The drag breakdown is discussed in Volume I, pp. 28-29 and in Supplement 4 to Volume I on p. 2. From the great number of papers concerned with evaluating the drag components one can easily see that the problem is an old one, the complete solution of which is yet unknown. Since drag is strictly a function of shape, velocity, and the fluid properties, obviously, then, the drag characteristics are the properties of a given type of airplane, and they must be evaluated separately for this type of airplane for simulation purposes. The lift, like drag, is a consequence of the body motion-velocity, and is given by

$$L = \frac{\rho}{2} V^2 c_L$$

where $c_L = m_o \alpha_o$.

When $\alpha_o$ = absolute angle of attack,

$$m_o = \frac{\frac{dc_L}{d\alpha}}{d\alpha}$$

and $c_e$ is characteristic of a given airfoil section. From (4) it is seen that $L \times V^2$ also, which greatly affects $F_z$ and constitutes a major part of it.

Velocity has a smaller influence on moments than on forces, and not much can be said quantitatively about them.

Investigating velocity components $u$, $v$, $w$, it is obvious that $V \approx u$ and the foregoing discussion applies also directly to $u$. Of $v$ and $w$, $v$ usually is more important due to its inherent aerodynamic significance: sideslip velocity. However, in certain periods of flight, $w$ has predominant significance, e.g., in landing and takeoff. Thus $F_y$ depends on $v$. There is also a strong effect of $v$ on the rolling moment $L$ if the wings have dihedral angle $\gamma$ (Figure 3).
Dihedral rolling is caused by differences in lift on the wings in sideslip. A pitching moment $M$ is also induced by the sideslip in the following fashion. When sideslip occurs, the tail unit is displaced laterally in the downwash field of the wings, causing a variation in the downwash along the span of the tailplane. This, in general, alters the pitching moment of the tailplane, which is a second order effect if the aircraft is completely symmetric, but not on that account always negligibly small.

In addition, $v$ has indirect influence on $N$ since sideslip, in general, creates a yawing moment tending to decrease the angle of sideslip. Consequently, this also reduces the angle of yaw and tends to bring the aircraft to an equilibrium position with zero sideslip.

The velocity component $w$ is generally very small and does not have any general characteristics similar to those of $v$. Although $F_z \approx w$, the influence of $w$ on $F_z$ is negligible, except in cases like pull-outs, steep turns, etc.

Another characteristic of highspeed aircraft that depends on $V^2$ is control reversal. A deflection of the flap, at an angle of incidence less than stall, in addition to an increase in lift and rolling moment, gives an increase in twisting moment tending to depress the leading edge and raise the trailing edge. This effectively causes a reduction in angle of incidence. If the wing is flexible, the angle of incidence is in fact reduced by an amount proportional to the applied twisting moment. Hence there is a reduction of lift and rolling moment caused by flap deflection, and the flap loses some of its effectiveness. This twisting moment for a given flap setting is proportional to $\rho V^2$. Therefore there is a critical speed at which the flap is totally ineffective, and above this speed the direction of its action is reversed.

2.3.2. Influence of $p, q, r$. The forces $F_x, F_y, F_z$, in general, are influenced by the variables $p, q, r$ only to a small, but not always negligible, extent. Equation (2) indicates which forces are affected for the symmetric case; for more general conditions (3) should be used.

The rolling rate $p = \dot{\phi}$ is certainly the most important of the three. Besides directly influencing $L$, it can appreciably change $F_y$ and $F_z$. As soon as one wing rises in roll, the forces on the aircraft cease to be in equilibrium, for weight and lift no longer balance; the result is a slight sideslip, with lateral and vertical deviations in the flight path.
However, as soon as the wing has acquired a rolling rate \( p \), an aerodynamic moment is created which opposes the rolling and is proportional to \( p \). This is due to the effective increase in the wing incidence on the downgoing wing and a corresponding decrease on the upgoing wing. The incidence change is inversely proportional to the flight velocity

\[
L' = k_p V_p
\]  \( (6) \)

\( k = \text{constant, with dimensions } L^4 \)

Investigating the effect of \( p, q, r \) on moments \( L, M, N \), it is apparent only that \( L, M, N \) are proportional to \( p, q, \) and \( r \), respectively. On the interaction of the variables on the moments, not much can be said in general. However, a specific case will point out the tendency. From experiments performed on a twin engined airplane, it follows that \( p \) and \( r \) have first order effects on \( L \) and \( N \) where \( M \) is affected only by \( q \), which result is in agreement with equation (2). The same terms also show velocity \( V \) as a first order effect on all the moments.

2.3.3. Influence of \( \delta_a, \delta_e, \delta_r \). The control surface deflections \( \delta_{\text{aileron}}, \delta_{\text{elevator}}, \delta_{\text{rudder}} \) are the principal devices besides thrust available to the pilot to control the aircraft flight. The control is indirect. Changing \( \delta_a, \delta_e, \delta_r \) changes the shape and therefore the effective angle of attack of lifting surfaces.

This causes moments \( L, M, N \); these cause \( p, q, r \), and finally through change in flight attitude introduce forces which produce the desired translation of the center of gravity.

This process corresponds to a time lag when the plane is represented by a linear transfer function.

It is noteworthy that \( \delta_e \) does not usually affect \( F_y, L, \) or \( N \). But the reverse, in general, is not necessarily true. The influence of the control surface deflections on \( F_x, F_z \) is small, and usually negligible, i.e., during 95% of flight time \( \delta_r \) and \( \delta_a \) practically do not have any influence, respectively, on \( F_z \) and \( F_x \) at all. For the rest of the flight time \( \delta_r \) and \( \delta_a \) can influence these forces only indirectly. Aileron deflections create roll and tend to cause yaw. Roll in some cases causes sideslip, and a small pitching moment is
induced. Yaw and pitch will influence $F_x$ to some extent, but these are second and third order effects and consequently very small.

2.3.4. Influence of $\alpha$, $z_e$, Mach Number. Velocity and angle of attack are direct causes of lift, and have considerable effect on aircraft drag and stalling characteristics. As an altitude parameter $z_e$ incorporates both density and temperature changes. Density affects the lift of the aircraft and is directly proportional to it (equation 4). The density is also an important parameter of air-breathing types of power plants. Temperature influences Mach number in the following fashion:

$$M = \frac{V}{a} = \sqrt[4]{\frac{RT}{49.8^2T}}$$

where $T$ is in degrees Rankine. Mach number seemingly does not influence any of the forces and moments. It appears at first glance that it represents simply a ratio of two velocities; but in aerodynamics and gasdynamics it is probably the most commonly used parameter. The following will illustrate the Mach number effect on lift and drag, $F_z$ and $F_x$. At a certain free-stream Mach number, the local Mach number at the point of minimum pressure on the airfoil reaches unity. This value of the free-stream Mach number is called the critical Mach number of the airfoil. Above the critical Mach number the flow is partly subsonic and partly supersonic. At points $M > 1$ a shock wave of increasing intensity becomes observable, where a rising drag coefficient accompanies the increasing shock intensity. Pressure discontinuity at the shock appears as a steep pressure gradient in the boundary layer. Thus for some Mach number above the critical the adverse pressure gradient causes separation of the flow, and a large rise in drag and decrease in lift occurs. The Mach number at which lift decreases, usually called the Mach number of divergence, is extremely difficult to predict theoretically.

As a consequence of the separation flow, stall occurs. In fact, stall is a direct consequence of a high angle of attack, and depends as well on boundary layer characteristics, i.e., on separation.

Lift is a direct result of velocity and angle of attack. In Figure 4a, common lift coefficient versus $\alpha$ curve is given. At $c_{L_{\text{max}}}$ the maximum angle of attack is reached; beyond that angle stall occurs, as lift falls off rapidly. Below a certain angle of attack the airfoil is essentially a streamlined body.
with a very thin boundary layer and with the separation point near the trailing edge; the turbulent wake is of course also thin, and the drag is small. At higher angles of attack the separation point moves forward, and the airfoil is no longer a streamlined body, but rather a bluff body with a large turbulent wake. The turbulent wake, which is very unstable, breaks up into oscillating waves. The fluctuating velocities in the wake are damped out as a result of viscosity and represent a loss in energy, which obviously increases the drag.

The nature of stall—whether it is smooth or abrupt—depends on the nature of the boundary layer and its behavior in the presence of the adverse pressure gradient. This pressure gradient has two effects: it causes transition from laminar to turbulent boundary layer, and it tends to cause separation of the boundary layer from the wing. At this point a new parameter, Reynolds number is introduced:

\[ \text{Re} = \frac{VL}{\mu} \]  \hspace{1cm} (8)

where \( L \) = a representative length.

At lower Reynolds numbers (\( \text{Re} < 300,000 \)) flow before and after separation is laminar. Due to its laminar nature the flow separation is mostly permanent. With increasing Reynolds number the transition occurs from laminar to turbulent boundary layer, and there is an increasing tendency for the separated flow to rejoin the surface. The turbulent boundary layer is better able to resist separation than the laminar, and as a result \( c_{L_{\text{max}}} \) increases with increasing Reynolds number.

Figure 4b shows the abrupt stall which occurs when the flow separates at the nose, resulting from a low \( \text{Re} \) (laminar boundary layer separation) and
resulting in a sudden increase in drag. Another noteworthy type of stall is shown in Figure 4c, in which an oscillation between nose stall (low Re) and readherence of the turbulent boundary layer (high Re) occurs that causes a shuddering of the airfoil at stall.

At stall attitude the forces and moments are unpredictable, and not much can be said about them.

2.4 Quantitative Results

In the preceding discussion no quantitative statements have been made, for the following reasons:

(1) Not much is known with accuracy sufficient for simulation purposes.

(2) Quantities differ from one airplane to another, and consequently their listing requires considerably more space than can be devoted here. They are available in other reports, however; in particular, Part 15 of this report encompasses many of these values.

(3) Finally, in almost all of the numerical results, the problem of the interactions of parts of an airplane has been omitted because of difficulties of both mathematical and physical nature.
3. CLASSICAL AERONAUTICAL ENGINEERING

3.1 The Engineering Approach

In the past, engineers have been able to build new airplanes without the complete force expressions needed for simulation, by a process of gradual evolution from existing models, using a combination of experience, empirical formulae, and shrewd guesswork. Although simulation needs something more concrete than engineering intuition, some of these approaches are described here, since they are helpful also for attacking problems in simulation, and illustrate the classical means of circumventing lack of knowledge, before simulation became available.

3.2 The Assumptions

The core of the engineering approach is the assumptions based on long experience, since a straightforward approach in engineering often proves futile.

3.2.1. Equations of Motion. For example, the equations of motion in six degrees of freedom cannot be solved by paper and pencil means. Since the solution of this set of equations is extremely important in aircraft design, engineers have assumed that the aircraft is symmetric about the xz plane for all flight conditions, which introduces an approximation, as Chapter 2 shows.

One reason for the convenience of this hypothesis is seen in the case of stability studies, where it is common practice to consider linearized perturbations from a fixed flight path. Only those functional dependencies which are general and odd (see equation 2) will appear when all but first order terms are neglected; the Taylor series expansions of the even functions begin with squared terms, which are ignored in the linear theory. This results in a separation between the so-called longitudinal (in the plane of symmetry) and lateral motion (see reference 13). Note that this is a linearized perturbation theory useful for stability studies; it can be shown mathematically that the only general (i.e., not linearized) motions for which this separation theorem is valid are trivial ones (reference 1). The wide application of the concept of longitudinal and lateral motion is due to the lack of knowledge of more general types of motion.

In fact, this method reduces the six equations to three equations for
longitudinal motion (plus one hinge moment equation), and to three equations (plus two hinge moment equations) for lateral motion.

3.2.2. Stability Derivatives. These equations are simultaneous homogeneous differential equations with constant coefficients. The constant coefficients consist of the airplane mass and inertia parameters and the so-called stability derivatives, which are the coefficients of the first order terms in the Taylor series expansion of the forces and are used to nondimensionalize the equations. As an example, two stability derivatives will be shown:

\[
\frac{\partial C_D}{\partial \alpha} = \text{the rate of change of the drag coefficient with angle of attack, obtained from the basic drag formula}
\]

\[
C_D = C_{D_{\text{friction}}} + \frac{C_L^2}{\pi e A}
\]

\[
\frac{\partial C_n}{\partial \dot{\theta}} = \text{the airplane's damping in pitch.}
\]

The usual practice is to evaluate the damping contribution of the horizontal tail, because it offers the largest contribution. This is estimated by determining the change in tail angle of attack due to the airplane's angular velocity \( \dot{\theta} \). The contribution of wings and fuselage is commonly accounted for by a multiplying factor of 1.10. From the arbitrariness of this procedure, it is evident why no attempt is made to measure stability derivatives to greater accuracy.

To discuss all of the dozen or so stability derivatives is beyond the scope of this report, but these two examples are representative of the remaining derivatives. It is obvious that the stability derivatives are not constant over a wide range of flight conditions, since, for example, \( C_D \) is different for the subsonic case and supersonic case. Furthermore, for a more rigorous treatment, the procedure of stability derivatives fails to give a sufficiently accurate answer. However, for the purposes of engineering design and stability analysis, the procedure has proved useful.

3.2.3. Additional Assumptions. If additional assumptions are introduced (e.g., constant velocity, constant altitude, controls fixed, or controls free), these equations reduce usually to three nondimensional equations. Experience has
shown that in this manner a great deal of useful qualitative information can be obtained about the airplane's motion.

The foregoing procedure assumes that only small disturbances exist, where there is no interaction between the symmetric and asymmetric degrees of freedom. Considering only small disturbances, no changes are introduced, because of the airplane's symmetry, in the external forces and moments outside the plane of symmetry. Conversely, small disturbances in roll, yaw, or sideslip do not affect the symmetric forces and moments.

As Chapter 2 indicates, this method does not apply for large disturbances, since in this event definite interaction exists between the symmetric and asymmetric forces and moments.

3.3 Step-by-Step Integration

Another method is step-by-step integration. At zero time \( u, v, w, \ldots \) are specified and the values of the air reactions are computed on the assumption that \( \dot{u}, \dot{v}, \dot{w}, \ldots \) remain constant over a small time interval. A solution can be obtained with any degree of accuracy required, provided the time interval is sufficiently small, but the process is extremely long and laborious and the final result is related only to the chosen initial conditions. In addition, the practical value of such results has been rather limited by lack of knowledge concerning the exact relationships between the airplane motions and the air reactions when the disturbance components are large.

3.4 The Separate Airplane Sections

In order to find the air reactions in flight, the engineering approach has been to divide the airplane into a number of sections and to analyze each section separately for as many dynamic conditions as the complexity of the problem admits.

Because the surface configuration of most of these sections (wings, fuselage, empennage, etc.) are rather complicated and do not permit a mathematical analysis suitable for the engineering purposes, they usually are replaced by bodies of simpler geometry (triangles, rectangles, cylinders, etc.).

Assuming that the principle of superposition is valid, the solutions obtained from the consideration of several separate sections are added together,
in order to describe the total aircraft.

This is the common and accepted procedure in aeronautical engineering. It necessarily neglects the interactions between airplane parts, i.e., the influence of the fuselage on the wings, etc. The problem of these interactions has been receiving attention recently, but the difficulties of both a mathematical and physical nature, as previously mentioned, are still immense.

3.5 Present Equations

For information and reference purposes, some of the fundamental equations presently used are presented in the following chart (Figure 5.) These equations are not universal. They give only velocity components on the surface of the body. However, combined with the equation of continuity, the equation of energy, and the equation of state, this set of flow equations allows one to obtain density, temperature, and pressure and leads to evaluation of the surface forces (lift, drag).

The chart indicates in what domains the flight seems not to be possible due to low lift characteristics and tries to point out the difficulties that may be encountered in other flight regions, for example, additional drag terms with increasing Mach number, shock interaction, etc. Also the appropriate equations at different altitudes are shown, since the equations depend on the properties of the medium, which particularly means altitude. Several phenomena are listed that must be taken into account with the boundary conditions in order to specify difficult flight conditions completely. Most of the phenomena listed involve discontinuities along boundaries in the fluid separating different types of flow; for example, at supersonic speed the shock wave is a boundary separating the flow into two regions of different analytic solutions. The phenomena introduce constraints on the system of equations, resulting in overspecified systems of partial differential equations whose solutions may be found only with extreme difficulty.

Summarizing: From the fundamental system of six equations one can obtain the following fundamental forces:

1. Friction drag
2. Lift
3. Induced drag.
Flight in this region is not feasible due to low lift characteristics.

Interaction between reflected and incoming molecules.

Interaction between shocks along the boundary layer and transition and separation.

Interaction between front shock and boundary layer.

Transition point which varies with thermodynamic and kinematic properties and which gives the percentage of surface covered by laminar and by turbulent flow.

Separation point of boundary layer.

Wave drag (front form drag)
Base drag

Friction drag
Induced drag
Basic

Figure 5
The next fundamental force, a part of "form drag," the so called "base pressure," is obtained from the fundamental system but with a constraint. In addition to phenomena listed in the chart that introduce constraints on the fundamental system, several more may be listed:

1. Interaction, wing-fuselage-tail.
2. Magnus effect.
3. Downwash effects.
4. Effects of distortion of lifting surfaces.
5. Effects of aero-elasticity.
6. Effects due to misalignment of nozzle.
7. Effects of formation of "near vacuum" conditions around the body.
8. Effects of gas imperfections.
9. Effects of surface irregularities, particularly due to high heat rate influence over a long period of time.

The values along both altitude and Mach number axes are only approximate since there exist no exact and sharp transition from one flight region to another. The equations are essentially approximations to the equations of motion where Euler equations can be considered as the first approximation, Navier-Stokes the second, and Grad's and Burnett's as the third approximation. These equations differ only by different stress tensors $P_{i,j}$ and heat flux vectors $S_i$ as can be seen below.

The hydrodynamic equation of motion:

$$u_{1,t} + u_r u_{1,r} + \rho^{-1} (p_i + p_{1j,j}) = 0 \quad \text{(ignoring body forces)}$$

Navier-Stokes:

$$P_{ij} = - \left[ (u_{1,j} + u_{j,1}) - \frac{2}{3} \mu u_{1,i} \delta_{ij} \right]$$

$$\frac{1}{2} S_i = q_i = - \lambda T_{1,i}$$
Grad:

\begin{equation}
\begin{aligned}
\rho_{ij,t} + \frac{1}{5} \left( S_{1,j} - S_{j,i} - \frac{2}{3} S_{r,r} \delta_{ij} \right) - \frac{2}{3} p_{rs} u_{r,s} \delta_{ij} + (p_{ij} u_{r},r) \\
+ p_{ij} u_{j,r} + p_{jr} u_{i,r} + p(u_{i,j} + u_{j,i} - \frac{2}{3} u_{r,r} \delta_{ij}) + \rho \rho_{ij} \mu^{-1} = 0
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
S_{i,t} + (S_{1} u_{r},r) + \frac{7}{5} S_{r} u_{i,r} + \frac{2}{5} S_{r} u_{r,i} + \frac{2}{5} S_{1} u_{r,r} \\
+ 2 \rho \rho_{i,r} + 7 \rho_{i} (RT)_{r} - 2 \rho^{-1} \rho_{i} (p_{rs} - p \delta_{rs})_{r} + 5 \rho (RT)_{i} \\
+ \frac{5}{2} \rho \rho_{S} \lambda^{-1} = 0
\end{aligned}
\end{equation}

Burnett:

**Stress Tensor**

\begin{equation}
\begin{aligned}
p_{ij} = -2 \mu e_{ij} + K_1 \mu \frac{u_{k,k}}{p} e_{ij} \\
+ K_2 \mu \frac{u_{k,k}}{p} \left[ -\frac{p_{i}}{p} e_{i,j} + \frac{u_{k,k} u_{j,k}}{p} - 2 \varepsilon_{i k} \varepsilon_{k j} \right] \\
+ K_3 \mu^{2} (\rho T)^{-1} \overline{T}_{i,j} + K_4 \mu^{2} (p \rho T)^{-1} \overline{T}_{i} \overline{T}_{j} \\
+ K_5 \mu^{2} (\rho T^{2})^{-1} \overline{T}_{i} \overline{T}_{i} + K_6 \mu^{2} p^{-1} \varepsilon_{i k} \varepsilon_{k j}
\end{aligned}
\end{equation}
where
\[ K_1 = \frac{4}{3} (\frac{7}{2} - T \mu^{-1} \mu, T) ; \]
\[ K_2 = 2 \]
\[ K_3 = 3 \]
\[ K_4 = 0 \]
\[ K_5 = 3 T \mu^{-1} \mu, T \]
\[ K_6 = 8 \]
\[ e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) - \frac{1}{3} u_{k,k} s_{ij} \]

Heat Flux Vector
\[ q_i = -\lambda T_{,i} + \theta_1 \mu^2 (\rho T)^{-1} u_{j,j} T_{,i} + \theta_2 \mu^2 (\rho T)^{-1} \frac{\partial}{\partial x_j} \]
\[ + 2 u_{j,i} T_{,j} \]
\[ + \theta_3 \mu^2 (\rho p)^{-1} p_{,j} + \theta_4 \mu^2 \rho^{-1} \frac{\partial}{\partial x_j} \]
\[ + \theta_5 \mu^2 (\rho T)^{-1} T_{,j} \] \[ e_{j1} \]
\[ (15) \]

where
\[ \theta_1 = \frac{15}{4} (\frac{7}{2} - T \mu^{-1} \mu, T) = \frac{45}{16} K_1 \]
\[ \theta_2 = -\frac{45}{8} \]
\[ \theta_3 = -3 \]
\[ \theta_4 = 3 \]
\[ \theta_5 = 3 (\frac{35}{4} + T \mu^{-1} \mu, T) \]
Equation of continuity
\[ \rho \frac{D}{Dt} \left( C_{v} T \right) + p u_i \cdot i + \frac{1}{2} \, S_{i,j} = 0 \]  \( (18) \)

where \( D \) denotes the operator
\[ \frac{D}{Dt} = \frac{\partial}{\partial t} + u_i \frac{\partial}{\partial x_i} \]

Standard tensor notation is employed in the above equations, including the summation conventions (i.e., repeated subscript indicates summation over that subscript), and the comma for partial differentiation. Thus \( p_{ij,j} \) represents expressions of three terms each:
\[ p_{ij,j} = \frac{\partial p_{ij}}{\partial x_1} + \frac{\partial p_{ij}}{\partial x_2} + \frac{\partial p_{ij}}{\partial x_3} , \quad i=1,2,3 . \]
4. THE ENGINEERING APPROACH AND SIMULATION

4.1 The Problem: Expressions for Forces and Moments

Simulation, like aeronautical engineering, is of necessity interested in concrete, detailed information and is confronted with the problem of solving the six equations of motion in their full complexity. (For example, it is not possible to consider the longitudinal and lateral motions separately in the general case.)

In order to solve the equations of motion, simulation engineers want to know the expressions for the forces $F_x$, $F_y$, $F_z$, and the moments $L$, $M$, and $N$. As pointed out in Chapter 1, these expressions are not known today, although, in general, the variables on which the forces depend are known.

4.2 Classical Considerations

Logically an aeronautical engineer may be called upon to find the expressions for the forces. Most likely he will take one look at equation (2) and say that it is not valid. Next he will draw a figure (Figure 6)

![Figure 6](image-url)
and write

\[ F_i = F_i (L, D, W, T) \]
\[ M_i = M_i (L, D, W, T) \]

(19)

where \( i = x, y, z \)
\( L = \text{lift} \)
\( D = \text{drag} \)
\( W = \text{weight} \)
\( T = \text{thrust} \)

These forces are the essence of classical aerodynamics, and they apply for both level flight and for maneuvering conditions. In principle equations (9) and (2) serve the same purpose, to describe the forces on an airplane, but from necessity expressions are used by engineers different from those of simulation.

4.3 Simulation vs. Aeronautical Engineering

As pointed out earlier, the engineers in order to facilitate simpler calculations have introduced a number of approximations and have expressed for simplification purposes the equations in terms of coefficients \( (C_L, C_D, C_N, \text{etc.}) \). Engineers also seek forces; but they want rather the characteristic modes of motion, in order to determine the nature of the control required to fly the airplane. The problem of simulation is to reproduce the complete behavior of the aircraft resulting from any given motion. This means that all of the forces (changed by the control surface motion) that ultimately cause a typical aircraft motion have to be introduced into the simulation network to give determinant equations of motion which can be solved for the trajectory. Nor is it sufficient to restrict the thrust to a constant value; a correlation between throttle action and standard thrust rating is necessary. Moreover, lift and drag values are definitely not constant over a wide range of flight conditions in which simulation is interested. The influence of the altitude parameter (density, temperature) must also be included, and the Mach number effect is often of considerable importance.

It is easily seen that in simulation the number of variables is larger than required for ordinary aeronautical engineering purposes, partly because of
rather drastic approximations and restrictions of generality often used by engineers and partly because of the different point of view taken in simulation. Certainly engineering experience should be utilized as a cornerstone for simulation. The problem now is to find and list all the information known concerning the forces on an aircraft. The next step is to translate the useful information from the engineering variables (L,D,T,V) into simulation terms, at which stage extreme care must be used to compensate for prior approximations. The last and largest problem is to combine the existing information into the necessary expressions for the forces and moments.

4.4 A New Method

The preceding sections show that the existing methods for solving the equations of motion fail to give realistic solutions without introducing some rather restrictive approximations. For the sake of clarity the problem is defined once more.

Simulation is interested in finding the trajectory for the general conditions of an aircraft flight. In order to find the trajectory, the differential equation \( F = ma \) must be solved for six degrees of motion, for each of which the expressions for \( F_x, \ldots, N \) must be known.

Explicit expressions for the forces are unknown, except under very restrictive conditions. Also, it is not feasible to try to solve for explicit expressions for the trajectory, i.e., \( x(t), y(t), z(t) \); the equations defining these expressions implicitly are too complicated to admit solutions for the explicit expressions. However, during the last 10 years a different approach has been developed to serve the same end.

The method is to calculate by means of simulation the trajectory without obtaining the precise equation for it. Essentially the simulator solves the differential equation \( ma = F \). One of the principal difficulties has been to prescribe expressions for \( F \) which represent the real forces adequately. Approximate expressions have been used successfully in particular investigations, such as stability analyses, but it now seems not only impractical to prescribe explicit expressions for general maneuvers, but in fact unnecessary.

Instead a new method which will take full advantage of the new techniques
available is being vigorously pursued. This proposition, though new to both aeronautics and simulation, is natural to each and should provide a tremendous impetus to each. Simply stated, it is proposed to determine forces not explicitly by algebraic expressions but implicitly from the more basic differential equations.

This may be accomplished in the following manner. From the basic physical hypotheses of fluid dynamics, precise differential equations are sought to define the desired forces implicitly. By means of a force simulator, the forces can be generated in the form of time-varying voltages and the results fed into a motion simulator, which simultaneously produces the trajectory. The motion simulator solves a set of differential equations (i.e., \( F = ma \)) and records the voltages representing \( x(t) \), \( y(t) \), \( z(t) \) as functions of time. That is, the inexact explicit algebraic equations used in the engineering approach are replaced by more exact implicit differential equations, and the curve is obtained without knowledge of the explicit expressions for the forces or the trajectory. A simulator handles differential equations as easily as algebraic equations, if not easier, since an analog computer integrates better than it multiplies. The advantage of introducing fewer approximations is obvious.

The goal of analytically obtaining algebraic expressions for the forces has been abandoned as impractical and unnecessary. If the differential equations defining the forces were known, the problem would be solved; but, except in the case of subsonic, low altitude flight of conventional aircraft, the required differential equations are still being developed from basic physical principles; the program in fluid dynamics leading to this goal is described in the following chapters.

Also in the next chapters is outlined the present and past status of fluid dynamics, aerodynamics, and thermodynamics. Deficiencies in past and present theories are discussed, and the point is strongly emphasized that new fundamentals are required in order to allow descriptions of the motion in all the regions and velocity ranges in which future flight will occur.
5. THE MODERN TREND IN FLUID DYNAMICS, AERODYNAMICS, GASDYNAMICS, AND THERMOAERODYNAMICS

The general field of fluid dynamics stands today at a turning point. Based now upon the fundamentals of the mechanics of continuum, it fails in such domains as hypersonics and slip flow. The necessity for building new fundamentals capable of supporting a reliable structure for the theory of motion in all the domains in which air vehicles may fly in the future is the most urgent task confronting this field. The following remarks apply to this problem.

5.1 Fundamentals of Fluid Dynamics

At its beginning the field of fluid dynamics was established basically as the field of liquid dynamics, i.e., hydrodynamics. In the course of time and development, the necessity arose for generalizing the laws of motion of gases, as well. This was simply accomplished by assuming that the density is variable. Occasionally additional assumptions have been made, that the coefficients of viscosity and heat conductivity are functions of temperature. In many problems of a laminar nature the equations so obtained give satisfactory information concerning the nature of the flow, but in the case of a turbulent motion these equations, of the type of Navier-Stokes, do not give satisfactory answers. Extension of the domain of applicability of these equations to supersonic and to the beginning of the hypersonic regions may prove to be an admissible and acceptable experiment, but already serious complications arise if one attempts to apply these equations to the hypersonic region, or to the slip flow region. It is now generally assumed that these equations do not adequately describe the status of the gas in those regions. Of course, still more serious complications appear in the domain of the highest atmosphere or the region of celestial and stellar fluid dynamics.

In general the Navier-Stokes equations are based on the mechanics of continuum. Since the fundamentals of the mechanics of continuum seem to break down at the threshold of the hypersonic region, new fundamentals must be established. The following possibilities may be considered, depending upon
the region in which the motion in question takes place:

1. kinetic theory of gases, in particular of polyatomic gases;
2. improved theory of free molecule flow and of the Newtonian gasdynamics;
3. magneto-hydrodynamics;
4. theory of relativity and the relativistic equation of fluid motion;
5. celestial fluid dynamics.

5.2 The Engineering Approach

The deficiencies in the existing theory of fluid dynamics are not the only ones which appear in practical applications of fluid dynamics and in the comparison of the theory with experimental data. Possible solutions are still more simplified in the so-called engineering approaches: linearized methods, similarity rules, various kinds of Prandtl-Glauert, Kármán-Tsien etc. rules, rules of thumb, various approximations, extrapolation methods from low Mach numbers to high Mach numbers, etc.

An approach is used to obtain the velocity pattern in the slip flow region which takes the ordinary Blasius solution and shifts the curve so obtained by a distance equal to the slip flow velocity at the surface of the solid body. It is recommended that the magnitude of the latter quantity be taken from test data. Although one can admire the ingenuity of such methods, it is certainly true that all such approaches cannot be included in the scientific part of fluid dynamics. They always remain in the domain of empirical and semi-analytical speculations, which work in certain cases and at certain Mach numbers. But definitely they cannot be considered as constructed on solid theoretical foundations and may break down unexpectedly in the least expected regions. Experience has corroborated these conclusions.

5.3 The Mathematical Approach

The effort in fluid dynamics to solve the practical problems by means of a fully mathematical approach is as old as the field itself. One need only refer to the classical hydrodynamics of the 19th century, various ingenious propositions in aerodynamics in this century, the whole approach to the boundary layer theory up to now, free boundary flow theory, etc. In some cases this approach is successful, but in others it has failed. In many cases the failure
is due to nonlinear boundary value problems, which, as is well known, do not possess any theory. But many such difficulties can be overcome by means of the modern high speed computing devices. Although they allow solutions only for particular numerical examples, nevertheless the solution of a great number of such numerical examples may help enormously in finding some form of a more general analytical solution.

5.4 The Modern Trend

From these facts, it seems obvious that the field of fluid dynamics stands today at a turning point. The necessity for obtaining data in such regions as hypersonics, Newtonian gas dynamics, and the celestial region requires a strong decision to construct new fundamentals based not upon the mechanics of continuum but upon much broader principles. These principles should include such fields as aeroballistics, the kinetic theory of polyatomic gases, magneto-hydrodynamics, free molecule flow, Newtonian gas dynamics, and even celestial fluid dynamics. It is obvious that an enormous problem, whose magnitude at the present time cannot be completely understood, is standing before this field.

5.5 Possible Improvements in Fluid Dynamics

It is generally assumed today that a theory of fluid dynamics based on the mechanics of continuum is inadequate and that the only possibility for high altitudes and high speeds is to refer to the kinetic theory of gases. In fact, the problem of building a theory of fluid dynamics based upon the fundamentals of the kinetic theory of monatomic and polyatomic gases seems to be the most urgent at the present time. The next problem of importance is that of a statistical fluid mechanics, which is difficult but certainly not impossible. The present theories of free molecule flow and of Newtonian gas dynamics also appear obsolete; it is questionable whether the existing model of the free molecule flow is as close to reality as had been hoped.

The problem of the interference of molecules in striking and being reflected from a solid body, which is of great importance for transport phenomena near the surface of a body, is unsolved, and certainly will not admit of an easy solution. Other problems involve magneto-hydrodynamics, celestial and stellar fluid dynamics, and the application of the theory of relativity to problems of a fluid dynamics nature. In the presence of large temperature gradients and high
velocities, differences in electron charges may appear, and subsequently affect the electromagnetic phenomena. Furthermore, in the regions of very high atmosphere the relativistic equations of motion of a fluid may have to be applied.

5.6 The Ideal Goal

In order to understand the situation better and to apply the considerations previously discussed to the fields of fluid dynamics, aerodynamics, gasdynamics, and thermoaeodynamics, one must begin with an examination of the field of aeroballistics.

The goal of this field is well known: to calculate the trajectory of an air vehicle flying at very high altitudes and very high speeds. The analytical approach to this problem is also very well known: starting from the dynamic equations of motion of a solid body, to find the velocity pattern and the trajectory. Since the dynamic equations of motion of a solid are an expression of Newton's law, one is confronted with the problem of knowing exactly all the forces acting upon the solid body in question. But the aerodynamic forces acting on a body moving in a fluid are functions of the velocity of the motion, angle of attack, etc. Hence basically the equations of motion are of a "nonlinear" character, and the only way possible to solve them is with some kind of limiting process. This requires a full knowledge, in the analytical meaning, of the functions of the (n-1) approximations in order to calculate the nth approximation. Of course it seems obvious that the approximative solutions may be presented in various forms: analytical, graphical, tabular, etc. Thus, at least theoretically, the "feeding" of the computing machine system to calculate successive approximations can be achieved in more than one way. But to assure continuity and smoothness in the calculation procedure, it seems necessary to present these functions in analytic form.

Thus one of the ultimate goals of aeroballistics seems to be clearly defined: given the geometric characteristic data of the air vehicle and the required range, to find the trajectory of the air vehicle, etc. Obviously a great variety of these problems exist, and attempts to achieve the goal were initiated almost simultaneously with the origin of the field of ballistics. But, as is known, ballistics has never reached the goal; all attempts in the past have been fruitless because of insurmountable mathematical difficulties. Until recently
ballistics has been an experimental science, but a thorough change in the point of view and in the attitude of ballisticians has taken place in recent years. Because of the great progress in the field of analytical aerodynamics, ballistics has returned to its previous standpoint and once more launches an attack upon the mathematical analysis of its own field.

In conjunction with long range missiles, the same task is being undertaken by aeroballistics. Naturally this attitude superimposes the ultimate, ideal goals upon the fields of fluid dynamics, aerodynamics, gasdynamics, and thermoaerodynamics. These are: given the geometric data of the air vehicle, to find all the aerodynamic characteristic coefficients, such as $C_{D\text{ total}}$, $C_L$, $C_M$, etc., which are necessary to calculate the data required in aeroballistics.

5.7 The Present Reality

The tools employed today in aerodynamics in finding the coefficients of drag and lift include

1. mathematical analysis,
2. wind tunnel tests,
3. flight tests,
4. ordinary firing range tests,
5. vehicle tests,
6. free air long range firing tests.

None gives complete results; the techniques complement each other. In spite of all the difficulties which are inherent in the first approach, it seems to offer promising prospects at the present time, at least in the preliminary stage of the the long range missile program. After long range missiles have flown enough to accumulate flight data, very likely the proposition of precise semi-experimental or semi-analytical formulas will be a fairly easy task.

5.7.1 Wind Tunnel Tests. The difficulties connected with the operation of wind tunnels at Mach numbers 7 to 10 are known, as well as the enormous difficulties connected with tests of the simplest possible kind at speeds of Mach 10 or above. It seems that the collection of the required data at Mach numbers 10 to 20, say, by wind tunnels tests is a distant prospect, and there is very little hope that preliminary calculations of the trajectories of long range guided missiles can be achieved on the basis of data from this source.
5.7.2 Flight Tests. Such tests are certainly powerful and reliable sources of information, although the domain of applicability of the data is restricted to the range of velocities and altitudes actually covered by the air vehicles in flights. As past experience has shown, the procedure of extrapolation of the data is risky.

5.7.3 Ordinary Firing Range Tests. These trial and error procedures over a distance of a few hundred yards are extremely expensive and, moreover, do not give good information on the detailed pressure and force distributions on the projectiles.

5.7.4 Vehicle Tests. The small number of tests performed by the use of vehicles, on straight, long railway tracks, does not permit an estimate of their value at the present time. But there is no reason to believe that they will prove more advantageous than the other test procedures.

5.7.5 Free Air Long Range Firing Tests. These tests, performed over a distance of 50 miles or so, are not free of the disadvantage of flying in the lower parts of the atmosphere, whereas the most important problems appear today in flights in the upper and highest parts of the atmosphere.

5.8 Mathematical Analysis

Mathematical analysis seems to present today the most hopeful possibilities. Although many problems in fluid dynamics do not possess existence and uniqueness proofs and their attainment can hardly be expected in the near future, nevertheless there is good reason to hope for the development of various techniques for the construction of solutions for many of the problems in question. The great possibilities offered by modern computing devices seem obvious; a great number of particular cases solved by means of such devices may indicate the proper direction of approach for proofs of existence or uniqueness of various kinds.

Today it is not possible to foresee which subfields in fluid dynamics, aerodynamics, gasdynamics, and thermoaerodynamics should or will be covered by the use of mathematical analysis. At the present time we are at the threshold of this approach, particularly when such auxiliary fields as the kinetic theory of gases as applied to the problems of fluid dynamics are considered.
The necessity for constructing a theory of fluid dynamics based on the kinetic theory of gases, both monatomic and polyatomic, is obvious. Moreover, the extension to the range of free molecule flow and Newtonian gas dynamics is so urgent that no discussion on the subject seems needed. Boundary conditions in the form of slip flow at the surface of a solid body moving in a gaseous medium and the heat phenomena at the surface require the analytic form of the problems in question. Hence the necessity for using simulation is indisputable and quite urgent.

Recently some work has been done on the application of the kinetic theory of monatomic gases to the theory of the boundary layer. Both versions are considered: Enskog-Chapman-Burnett system and Grad's proposition. The system of equations obtained for a two-dimensional boundary layer is complicated but solvable. This proves that the fluid dynamics equations based on the kinetic theory of gases are solvable if adequate computing facilities are available. The other approaches possible, mentioned in section 5.1, i.e., statistical theory of fluid dynamics and some new developments in free molecule flow, will also certainly require large simulation facilities.

5.9 Development of Modern Fluid Dynamics

5.9.1 Phase I. From the previous discussion it seems obvious that modern fluid dynamics is at the beginning of phase I of its development. In this exploration phase rough laws are sought that apply to the situation under investigations, i.e., the laws governing the behavior of the characteristic aerodynamic coefficients of a flying air vehicle at all possible altitudes, velocities, gas rarefaction conditions, etc. The attempt is to identify the basic parameters and their ranges and to develop the methods of approach. Preliminary calculations are performed, such as those applying the kinetic theory of monatomic gases to describe a motion of a solid body in a polyatomic gas. A solution is proposed for a complicated system of the boundary layer motion based on the kinetic theory of gases without having any mathematical theory of the boundary layer. Many attempts are made to solve nonlinear boundary value problems, although a theory of nonlinear boundary value problems does not exist, particularly with nonlinear boundary conditions.

WADC TR 54-250, Part 16

33
5.9.2 Phase II: the Proofing-In Phase. This is a phase where less expendable material will be used, current materials used more sparingly, improvement of techniques made, all parameters and all laws checked, and all factors modified to fit current, precise requirements.

5.9.3 Phase III: the Simulation Phase. At phase III one should have confidence in the results of the first two phases and be ready to undertake simulation so that full scale experiments no longer need be made, although not all aspects of all problems may be simulated. Equipment used in phase I is obsolete. For further proofs of the laws involved and for exploration of the effects of variations in fundamental parameters, many "phantom" runs can be made.

5.9.4 Phase IV: the Development Phase. One should at this stage be able to predict and to plan future weapons, making use of what was learned in previous phases concerning the parameters, laws, and physical systems involved. At this stage new designs should be accomplished with minimum waste.

5.10 Final Remarks

Some problems connected with the future development of the field of fluid dynamics have been discussed. To those already mentioned, some others may be added:

(1) In the theory of the boundary layer is the interesting problem of establishing a rigorous mathematical theory, providing various existence proofs, slip flow velocity, etc.

(2) In the theory of airfoils the linearized methods prove inadequate in the range of the supersonic flow region. Such problems as the downwash theory necessarily require the use of nonlinear equations.

(3) The shock theory in viscous, heat conducting gases is far from complete. In spite of a few encouraging solutions and propositions at moderate Mach numbers, no solutions at high or very high Mach numbers are known. The problem of shocks in the hypersonic region, where the shock is very thick and presents rather a transition region than a discontinuity surface, is completely untouched.

(4) The problem of the interaction between body and lifting surfaces, of less importance in the subsonic range where the aspect ratio of the lifting surface is large, is important in modern air vehicles, where the ratio of the lifting surfaces is small.
(5) The field of magneto-hydrodynamics is, truly speaking, not yet established.

(6) The fields of free molecule flow and Newtonian gas dynamics apply very crude, first order approximation methods, which sometimes are illogical and hard to justify. Much more work must be done in these fields before a clear picture of the phenomena can be presented.

(7) Practically and theoretically nothing is known about the relativistic approach to problems of celestial and inter-space fluid dynamics.
6. THE KINETIC THEORY OF GASES AND
STATISTICAL FLUID DYNAMICS

These remarks are of an informative nature; lack of space does not allow a very thorough discussion of all the phases of the fields in question.

6.1 Kinetic Theory of Gases

The aim of statistical mechanics is to derive the macroscopic properties of matter from the microscopic properties of molecules. The term includes, according to recent practice, nonequilibrium as well as equilibrium states. Also, insofar as it is possible, the attempt is to derive the results of macroscopic observation rather than merely show that they exhibit a logical structure similar to that of some arbitrarily chosen model. In equilibrium the classical theory of statistical mechanics gives an essentially complete answer, but much still remains to be done in nonequilibrium. As is well known, statistical mechanics and thermodynamics are conventionally based on energy concepts, and fluid dynamics is based on momentum as well as energy; while the other integrals of classical particle dynamics, notably the angular momentum, are usually ignored or minimized. Some interesting results of the generalized theory, including a complete set of integrals, were recently obtained by Grad. The present remarks are largely based on Grad's works, who has found it possible to give a more rational presentation to the whole of classical statistical mechanics.

These remarks are limited to a discussion of the behavior of rarefied, monatomic gases. A careful analysis of the derivation of the Boltzmann equation shows that the following assumptions are used: (1) point molecules, (2) complete collisions, (3) a slowly varying distribution function, F, and (4) molecular chaos.

The first assumption states that there exists a time interval dt which is large compared to the average duration of a collision, $\tau^*$, but is nevertheless small compared to the average time between collisions, $\tau$; $\tau^* \ll dt \ll \tau$. The third assumption, which is of interest in fluid dynamics, is that F does not vary appreciably over the distance covered by the average molecule in the time dt.
This restricts $F$ to be essentially constant over a distance comparable to the size of a molecule, but imposes no restriction on the variation in $F$ over distances comparable to the mean free path. In particular, flows around objects of molecular size cannot be handled by the Boltzmann equation, but object dimensions of the order of a mean free path cause no difficulty. The assumption of molecular chaos has special significance, for it cannot be given a simple physical interpretation. Without this assumption, no determined equation for $F$ can result. The molecular chaos assumption has been proved only for the equilibrium state. Intuitively, any correlation between the velocities of two molecules which are not in each other's field would seem to come from previous collisions between the two molecules, previous collisions of each with a third molecule, etc. This suggests that in a rarefied gas in which collisions occur infrequently the molecular chaos approximation is accurate. Also, since it has been proved for equilibrium, it can be expected to be accurate near equilibrium. Why molecular chaos must be assumed in the derivation of the Boltzmann equation is discussed in Grad's works.

It seems obvious from the nature of the assumptions underlying Boltzmann's equation that one may expect some deviation between the solutions of this equation and the actual status of a gas in nonequilibrium states. Thus the question of what kind of nonequilibrium states exist to which this equation can be applied is more or less open.

The state of a physical system is given a rather flexible meaning in practice, usually including only whatever information about the system is thought useful for the purpose at hand. Normally the state is determined by the values of certain state variables. Depending on what variables are selected, more or less information can be given about the system; in other words, it is possible to describe a physical system on various levels of information. To describe a nonequilibrium state satisfactorily on any level, equations must be known which determine the variation in time of the variables of state once their initial values are given. A solution of such a system of equations should be able to describe adequately the nonequilibrium state even if the variables are subject to rapid changes. The usual notion of the distribution function in the kinetic theory of gases should suffice for nonrotating spherically symmetrical
molecules with only one internal state. When only one kind of molecule is present, the distribution function, \( F \), approximately satisfies the Boltzmann equation.

Works on the Boltzmann equation by Hilbert, Enskog, Chapman, Burnett, and others result in solutions of that equation which are thermodynamic approximations, starting from the Boltzmann equation. In the first approximation, the left hand side of the Boltzmann equation is omitted, resulting in a nonlinear integral equation, the solution of which is a Maxwellian distribution function. The higher approximations satisfy inhomogeneous linear integral equations. As might be expected, this procedure leads to an asymptotic solution, since the derivatives of the unknown functions are neglected in each approximation. The first approximation leads to the Euler equations for a compressible, inviscid, non-heat-conducting fluid flow; the second approximation yields the Navier-Stokes equations; and the third approximation yields the so-called equation of slip flow or Burnett's equations. The polynomials used in the latter approach are numerical multiples of Sonine's polynomials, which arise in the study of Bessel functions.

The most promising solution of the Boltzmann equation has recently been proposed by Grad, who uses Hermite's polynomials. This solution may be considered as standing between the thermodynamic and the full use of the Boltzmann equation. From the few known solutions of the Boltzmann equation it seems probable that the type of analysis proposed by Grad is preferable to the Hilbert-Enskog method when considering rapidly changing flows, for example the internal structure of a shock wave.

The deficiencies of the kinetic theory of gases are known: applicability only to monatomic gases at the present moment. Although some attempts have been made to generalize the theory to polyatomic gases, the effort has not resulted yet in any applicable aspects of the problem. Kinetic theory is valid only for the equation of state of a perfect gas.

The question of the necessity of considering real gases, i.e., of taking into account the equation of state of a real gas is also very urgent. The influences of gas imperfections and the deviations from the perfect gas conditions in a real gas can be considered in three groups:

(1) Conditions outside the molecule which can be taken into account by a
adopting a more complicated equation of state in place of the perfect gas equation.

(2) Changes inside the molecule which are associated with the high temperatures. They arise from the fact that air is almost entirely composed of diatomic molecules of nitrogen and oxygen, so that in addition to the energies of translation and rotation, energy can be absorbed in the vibration of the atoms in the molecule. At high temperatures this energy is considerable. It is necessary, therefore, to consider a gas in which the specific heat ratio is variable across a shock wave or through a boundary layer. The process of internal changes is rendered still more complicated by the phenomenon known as heat capacity or relaxation time, and, in addition, as the temperature increases with increasing speed, the phenomenon of dissociation becomes apparent.

(3) Changes inside the atom indicator. It is possible that ionization of the atoms of the gas results from collision at the highest velocities. This effect becomes significant only at very high speeds, around Mach 20. In recent observations of meteor trails, and particularly in radar observations, there is evidence of considerable ionization of the air.

It is clear from this survey that the properties of a missile traveling at a very high speed through the air may be quite different from those expected on the basis of perfect gas properties.

6.2 Statistical Mechanics of Fluids

Turbulence has adopted the language of statistics. In the purely descriptive theory, the kinematics of a turbulent velocity field, the stationary random function seems to be a really well suited tool; it appears that a fairly good theory can be built on that ground; but in the dynamics of turbulence the results are meagre. The idea of a turbulent flow has compelled the employment of some of the methods of probability. Having introduced these methods, the attempt is made to take into account some equations suggested by fluid dynamics; the resultant mixture of probability and dynamics is not a very fortunate one. Thus the question of a statistical fluid mechanics arises.

The main characteristics of the statistical mechanics of a conservative dynamic system with a finite number of degrees of freedom and satisfying the Hamilton-Jacobi equations are the following:
(1) definition of the phase-space $\Omega$; every state of the system is characterized by a point $\omega \in \Omega$;

(2) proof of a uniqueness theory; starting at the initial time $t = 0$ from a given initial state $\omega$, all the subsequent or prior states $T_t \omega$, where $t$ varies from $-\infty$ to $+\infty$, are perfectly well determined and describe in $\Omega$ a curve, the trajectory $\Gamma(\omega)$;

(3) definition of a measure $\mu$ in $\Omega$, invariant under the transformations $T_t \omega$;

(4) proof of the ergodic theory: the time average computed along a trajectory $\Gamma(\omega)$ exists for almost all $\omega$; time-average and statistical-average are equal if the transformation $T_t \omega$ of $\Omega$ into itself is metrically transitive.

A fluid dynamics system does not have a finite number of degrees of freedom; hence we must take as phase-space $\Omega$ a function space. From the Lagrangian point of view, the state of a fluid, at a given time $t$, is defined by the position and velocity of all the particles. Accordingly, we take as phase space $\Omega$ the function space in which a point $\omega$ is defined by a set of three functions of three variables $(x_i)$ $(i = 1, 2, 3)$:

$$\omega = J_j (x), \quad (j = 1, 2, 3).$$

The successive states of the fluid shall be thus represented in $\Omega$ by the set of points $\omega$ (trajectory $\Gamma$) such that:

$$J_j (x) = u_j (x,t).$$

To proceed further we must now have a uniqueness theory: starting from a given initial state $\omega \in \Omega$ at time $t = 0$, are all the states or at least all the subsequent states of the fluid fully determined? Here difficulties are met. For an incompressible viscous fluid filling the whole space $X$, J. Leray has proved that, corresponding to initial conditions regular enough, there is one and only one regular motion during a finite interval; but the possibility is open that the regularity will break down after this finite time. Moreover, the results of Leray, as fundamental as they are for fluid dynamics, have been established for a case which is very far from the flows in which we are largely interested: Leray assumes that the total kinetic energy of the fluid is finite, an assumption essential in his work, but in most cases in fluid dynamics the kinetic energy of the system is infinite.
These difficulties are serious. But, of course, development of the statistical mechanics is not finished, and it is impossible to predict what results the future will bring.
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